An Economic Model of the Ethiopian Farm Household*
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Abstract

This paper presents a simple model of an Ethiopian farm household which captures three important aspects of the policy regime characterising the *Derg* period. These aspects are compulsory grain delivery, rationing in manufactured consumer goods, and rationing in modern farm inputs. The model involves two main innovations within the agricultural household modelling framework. First, a new procedure of analyzing the impact of the policy of forced grain procurement is introduced. The procedure enables us to directly characterize the effects of that policy on farm households' welfare, as well as the production and consumption choices they make. Second, it pulls together various strands of the relevant literature in a simple manner. In particular, it provides a more direct way of determining the welfare effects of rationing, compulsory grain delivery, prices, and incomes. Both individual and joint effects can be handled this way. The comparative static properties of the model directly show that forced grain procurement by the state reduces the welfare of farm households and distorts their production choices. The results also indicate that shortages (rationing) in manufactured consumer goods and modern inputs make commodity demands, input demands, and output supplies less price responsive.

1 Introduction

This paper presents a simple model of an Ethiopian farm household which incorporates a number of stylized facts mostly applicable to the 1980s. It is developed with the objective of examining the production, consumption, and welfare effects of forced grain procurement by the state, and shortages of manufactured consumer goods and modern inputs. The model involves two main innovations. First, a new procedure of analyzing the impact of the policy of forced grain procurement is introduced. The procedure enables us to directly characterize the effects of that policy on farm households' welfare, as well as the production and consumption choices they make. Second,

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it pulls together various strands of the relevant literature in a simple manner. In particular, it provides a more direct way of determining the welfare effects of rationing, CGD, prices, and incomes. Both individual and joint effects can be handled this way.

As a systematic study of the choices and constraints of Ethiopian farm households during the *Derg* period, this paper hopes to contribute towards a better understanding of the behaviour of such households in a number of ways[1]. Such study helps to identify the determinants of farm households' welfare and efficiency. It also promotes a more accurate characterisation of the pattern of responses of farm households to changes in market variables, production technology and government policies. In so doing, it provides some explanations regarding observed responses of such households and related outcomes in the agricultural sector. Furthermore, such analytical effort potentially leads to better empirical analysis, in general, and more accurate estimates of demand and supply elasticities, in particular. All of these are desirable in any context, but they assume greater significance in the light of the economic reform process Ethiopia has been undergoing since 1990. Specifically, some of the potential effects of this process can be pinpointed.

The paper is organized as follows. The rest of Section 1 motivates the modelling strategy adopted by identifying the main features of Ethiopian households and their operational milieu during the *Derg* period. It also describes aspects of that modelling strategy. Section 2 presents the model in some detail. The potential effect of compulsory grain delivery on the allocative efficiency of farm households is dealt with in Section 3. The fourth section focuses on the impact of changes in the level of forced grain procurement, ration levels and prices on household demands, supplies, and welfare. Section 5 concludes.

1.1 Motivation and characterization

In a brief and stylized characterization of a typical Ethiopian farm household during the *Derg* period, we can identify the following features[2,3]. During that period, such

[1] An auxiliary objective of the paper is to encourage a more wide-spread use of agricultural household models and the virtual price approach in studying Ethiopian farm households. As a result, it contains a rather detailed discussion of modelling techniques. We apologize to those who are familiar with these techniques.

[2] For a detailed discussion of these features see Taffesse (1997).

[3] A brief note about the institutional setup is in order. The peasant association (PA) and the service cooperative (SC) were at the centre of the local institutional environment of Ethiopian farming during most of the 1970s and the whole of the 1980s [Alemneh (1987), Taye (1992)]. Both evolved owing to the land reform of the mid-1970's.

A PA is established as a permanent association of farm household heads living within an area not exceeding 800 hectares. By 1982 there were about 20,000 such associations throughout the country [Tegegne and Tennassie (1984)]. Initially, the primary role of a PA was to implement the provisions of the Rural Land Reform Proclamation of 1975, in general, and land redistribution among its members, in particular. Subsequently, however, it assumed the role of the lowest administrative unit of the state in terms of assessing and collecting taxes, maintaining law and order, and locally coordinating and enforcing compulsory grain delivery. The effectiveness of the PA in these and related functions largely stems from the fact that, apart from residence in an agricultural locality (typically a village), a farming household can exercise the right to access to land if and only if its head is a member of the local PA.

A SC, on the other hand, is formed by a group of neighbouring PAs. Three to eight such PAs may belong to a SC. By the middle of the 1980s, there were about 3,800 SCs in the country [Tegegne and Tennassie (1984)]. SCs are established to facilitate the marketing of the output of farm households, to purchase and distribute modern inputs and manufactured consumer goods, and to promote the provision of services including health, educational, and milling services. The first of these objectives a farm household[4]:

(1) simultaneously produces several crops, cereals being the most important;

(2) operates a small land-holding allotted by the state on usufruct, and consisting of several variously endowed and located plots;

(3) employs a traditional technology of production with some application of improved inputs (the main ones being fertilizers);

(4) obtained its fertilizer supply from a government agency in the form of rations at subsidized prices, but appeared to be unable to fully satisfy its demand for this input;

(5) relies on family labour, augmented by traditional labour exchange relations and participation in the labour market;

(6) consumes part of its output;

(7) sold part of its output to the government, through the compulsory grain delivery (CGD) system, and part of it on the 'free' or 'open' market;

(8) bought manufactured consumer goods, primarily from the public sector, in the form of rations from a Service Cooperative's shop, but appeared to be unable to fully satisfy its demand for some of these goods.

1.1.1 compulsory grain delivery

A central feature of crop markets in the 1980's was the extensive intervention by the state. In fact the government of the day established an agency for this and related purposes. This agency is called the Ethiopian Agricultural Marketing Corporation (EAMC). Specifically, the EAMC was set up in 1976 with the following objectives [Fasil and Alemayehu (1985)]:

(1) to stabilize producer and consumer prices of grains;

(2) to encourage grain production through price incentives; and

(3) to ensure an adequate food supply for the public distribution system.

Initially, the Corporation's role in terms of its stated objectives was minimal. In particular, it was not successful in its task of purchasing produce from farm households. Partly in response to that failure, the government introduced the compulsory grain delivery (CGD) system in 1980. The system had three main features. First, farm households were required to sell a portion of their output to the EAMC at fixed, pan-territorial prices. After meeting this obligation (commonly referred to as the 'quota'), households were allowed to sell their grain to private traders or directly to consumers, usually on local, 'open' or 'free' markets. By 1986, it was claimed that

translated into coordinating the compulsory grain procurement from farm households. SCs also appeared to succeed in performing the second task [Tegegne and Tennassie (1984)].

[4] In this regard, all of these, with the exception of (4) and (7)-(8), apply to the late 1970's as well. Moreover, it is likely that (1)-(3) and (5)-(6) will hold for sometime in to the future.

Table 1: **Average Market and Procurement Crop Prices (1981-90)**

Price	*Barley*	*Maize*	*Sorghum*	*Teff*	*Wheat*
Market	54.3 (19.2)	47.6 (19.4)	52.6 (21.6)	74.8 (25.0)	65.4 (20.2
Procurement	28.2 (1.93)	20.0 (1.83)	23.8 (1.34)	39.3 (2.21)	31.6 (0.97)

NOTES: Computed from the information in EAMC (1987), CSA Statistical Bulletin Nos. 44, 59, 65, 85, 95, 105. The market prices are national averages. Standard deviations in parantheses.

EAMC had captured 30-40 per cent of the marketed surplus of grain in the country via CGD [EAMC (1987)].

Second, CGD involved the determination of both prices and region-level procurement targets (both the types of grain and the corresponding quantities) by the central government. In principle, the targets were set according to the productive potential of regions. The procurement plans were then distributed down the administrative chain. Each region allocated its target among subregions, and so on down to the level of the PA. Each PA was expected to determine the 'quota' of a farm household by considering farm size, total output, seed and own-consumption requirements [Befekadu and Tesfaye (1990)].

Third, contrary to one of its stipulated objectives (the second in the list above), the EAMC persistently paid very low prices for the grain of farmers. Table 1 reports the procurement prices of the five major cereals alongside their market prices. The latter are more than twice as high as the former for maize, sorghum and wheat. The differences are only slightly lower in the case of barley and *Teff*. The loss of income implied by the price differentials is likely to have been detrimental to the welfare of farm households. In addition, the weakening of incentives induced in this manner is likely to have affected the production choices of farm households.

It can be inferred from the above that CGD affects the welfare of producers. It may also affect their resource allocation decisions. In principle, the impact of the compulsory delivery system can be modelled in different ways. That it is an implicit form of taxation (or rent) seems to be the common view[5]. Thus, identifying an equivalent form of explicit taxation facilitates analysis. Azam (1992) proposes that the 'quota' is equivalent to a lump sum tax, and proceeds to examine its impact accordingly in a model where consumption and production decisions are separable. Consequently, the 'quota' can only affect the consumption decisions of farm households. This contrasts with the commonly made argument that the 'quota' system affects the production decisions of farmers [Franzel *et al* (1989), Schiff (1993), Schiff and Valdes (1994)]. It also does not accurately reflect the process of 'quota' allocation to peasant households. The most common criteria used in this process are[6]:

i) the potential crop output of the household; and

[5] See Taffesse (1989), Franzel et al (1989), Azam (1992), Dercon (1994), and Pickett (1993). This view is shared by those who analysed the procurement system in general, as well as its specific applications - see Sah and Stiglitz (1992), and Sah and Sirnivasan (1987).

[6] To the extent that it was not based on a 'quota schedule', the determination of 'quota' levels to be delivered by households was not uniform. Nevertheless, the most common practice was the imposition of relatively higher 'quotas' on households with higher outputs [see Alemayehu (1987)].

ii) the wealth (or, more precisely, the overall income-generating capacity) of the household, measured by some variables, including, size of land-holding, number of oxen and other livestock owned, and nonfarm income.

In the light of the above, it is argued that the 'quota' should be viewed as an implicit proportional output tax. One way of modelling this is to consider the 'quota' as a proportion of output, such that the revenue of the household from the crops(s) subject to CGD is summarized as[7]:

$$p_d^s \alpha Q_{ad} + p_d^m (Q_{ad} - \alpha Q_{ad}) = [\alpha p_d^s + (1-\alpha) p_d^m] Q_{ad} \tag{1.1}$$

where: p_d^s = the procurement (or EAMC) price of the crop subject to CGD; p_d^m = the 'free' or 'open' market price of the crop subject to CGD; α = the proportion of the 'quota' out of the total output of the crop subject to CGD; Q_{ad} = the total output of the crop subject to CGD. Multiplying the right-hand-side by $\frac{p_d^m}{p_d^m}$ and defining the implicit output tax rate as, $\rho = [(1 - \frac{p_d^s}{p_d^m})\alpha]$, a more compact expression of the impact of CGD on household revenue is derived:

$$\begin{aligned} [\alpha p_d^s + (1-\alpha) p_d^m] Q_{ad} &= p_d^m (1-\rho) Q_{ad} \\ &= p_d Q_{ad} \end{aligned} \tag{1.2}$$

where p_d represents the weighted average price of the crop subject to CGD. The average price defined this way represents the household's marginal value of a unit of output. Note that, since p_d^s is less than p_d^m, the valuation of Q_{ad} by p_d captures the tax nature of the 'quota'. Furthermore, observe that if a given 'quota' level is uniformly imposed on all households, the suggested procedure is inappropriate and that of Azam (1992) is the correct one.

1.1.2 rationing in manufactured consumer goods

In the study period, farm households had two sources of manufactured consumer goods. The first was the Ethiopian Domestic Distribution Corporation (EDDC). This government agency was entrusted with the task of distributing most manufactured goods to urban and rural consumers, government institutions, and private traders. EDDC supplied manufactured consumer goods (MCGs) to SCs, which subsequently ration what is available among PA members, mainly according to family size and composition. In this regard, SCs, supplied by EDDC, constituted the primary source of MCGs to peasant farmers. Sixty-nine per cent of a sample of farm households in Arssi, a relatively prosperous region, identified the SC as such [Alemneh (1987)].

The second source was the 'free' market, in which private traders sold consumer goods at higher prices. The private traders were largely supplied by EDDC. In addition, there is anecdotal evidence that this market served as a place for re-trading in rationed goods. In short, local markets served as a secondary market.

There is no direct evidence regarding how satisfactory the availability of MCGs was to peasant households. However, the following may shed some light:

[7] The analysis below does not consider ways other than adjusting crop-mix that households may have devised to avoid delivering the quota or minimize its impact. Thus α has to be viewed as relating to the implicit output tax actually paid by farm households.

1. During the 1980s, the peasant sector was, on average, allocated 16 per cent of the total value of EDDC supply. Similarly, during 1979/80-1983/84 the sector accounted only for 14 and 2 per cent of total salt and sugar supplied by EDDC, respectively[8]. Yet, the peasant sector accounted for more than 80 per cent of the population and about 40 per cent of the gross domestic product of the country. Moreover, those shares may overstate the actual availability of the goods to farm households, because some portion of these shares may have flowed back into urban centres, due to corruption.

2. As a matter of government policy, importation of MCGs was highly restricted. Indeed, the 1980's are associated with overall shortages of such goods.

3. In the context of shortages, private traders are likely to have preferred urban centres to rural markets, mainly owing to the relatively high costs of transportation involved in supplying the latter.

Hence, it is more than likely that farm households have faced shortages in MCGs during the 1980's. Therefore, two major features relating to the supply of MCGs can be identified - rationing in the form of an upper bound to how much a farm household can buy from a SC's shop, and overall shortages. Moreover, given the various constraints on private trade noted above, the role of local markets as a secondary market is unlikely to apply to all rationed goods, all localities, or all households. In principle, therefore, two classes of rationed consumer goods can be identified - those for which there is a secondary market and those for which such a market was absent. In the spirit of the usage of the terms by Sah (1987), these classes are respectively referred to as 'convertible' and 'non-convertible' rationed manufactured consumer goods[9].

Accordingly the impact of shortages in manufactured consumer goods is modelled in two ways. 'Non-convertible' rations determine the maximum amount of household consumption of the goods to which they apply. They can thus be accommodated by introducing a quantity constraint in terms of such goods in the households optimization problem[10]:

$$X_{Mn} \leq R_{Mn} \tag{1.3}$$

where X_{Mn} and R_{Mn} represent household demand for, and the ration level of, 'non-convertible' rationed manufactured consumer goods, respectively.

As to 'convertible' rations the possibility of trading in the secondary market means households are not restricted to consuming their rations. Households whose demand (X_{Mc}) is less than level of their ration (R_{Mc}) will sell the surplus on the secondary market. In contrast, households whose demand exceeds their ration, attempt to fill the gap through purchases on the secondary market. Assuming the secondary market clears, and clears at a price higher than the price at which the household purchases 'convertible' rations from the SC, the ration is a source of income for the first group of households, while it provides subsidized consumption for both groups. These effects

[8] All the supply shares reported in this paragraph are computed using data compiled from EDDC, *Statistical Abstracts*, Nos. 1-3.

[9] Sah (1987) compares four systems of distribution of consumer goods - nonintervention (pure market); nonconvertible rations; convertible rations; and queues. In the present case, the distinction between 'convertible' and 'nonconvertible' rations is embedded in a more general problem of explicitly modelling the production and consumption decisions of a farm household under rationing.

[10] See Neary and Roberts (1980), Deaton and Muellbauer (1980), Deaton (1981), Neary (1987), Cornes (1992), and Mckay and Taffesse (1994).

can be expressed by writing the expenditure on 'convertible' rationed manufactured consumer goods as:

$$p_c^s R_{Mc} + p_c^m (X_{Mc} - R_{Mc}) = p_c^m X_{Mc} - (p_c^m - p_c^s) R_{Mc} \tag{1.5}$$

where: p_c^m = the (secondary) market price of 'convertible' rationed manufactured consumer goods; p_c^s = the price at which the household purchases 'convertible' rationed manufactured consumer goods from the SC. The second term on the right-hand-side captures the subsidy/income generated by 'convertible' rations. The budget constraint of the household is adjusted accordingly.

1.1.3 rationing in fertilizers

Ethiopian farm households employ essentially traditional farming practices [Yeraswork *et al.* (1983), Pickett (1991), and Taye (1992)]. It is traditional in the sense that most of the instruments and methods were in use for centuries without major innovation. Land, labour, and oxen are the prominent factors of production[11]. Gradually, however, modern inputs are taking hold. This is particularly true of fertilizers[12].

Fertilizers constitute a major modern input which are increasingly used by farm households. This is particularly true of the grain surplus regions of the country. In the 1980s, fertilizers were distributed to farmers at subsidized prices. During 1977-1983, the Ethiopian Agricultural Marketing Corporation (EAMC) was the only agency responsible for the purchase and subsequent distribution of fertilizers in the country. In 1983, these functions were taken over by the Ethiopian Agricultural Inputs Supply Corporation (EAISCO). Both agencies distributed fertilizers to farm households via SCs. Each SC was entrusted with the task of allocating the available supply to its members in the form of rations. The level of this ration defined an upper bound to household purchase. Given the monopoly position of the supplier, the rations also set the upper bound for use. In this regard, the quantity of fertilizers supplied to the private peasant sector was very low. In fact, excess demand has been a regular feature of the market for fertilizers. [Franzel, Colburn, and Degu (1989)].

Fertilizers were distributed through the SC in the form of rations. For the purpose of analysis they are treated as 'non-convertible' rations. The ration level thus defines an upper bound to household purchase as well as use. Accordingly, rationing in fertilizers is introduced via the following additional constraint:

$$V \leq R_v \tag{1.6}$$

where: V and R_v represent household demand for and the ration level of fertilizers, respectively.

1.1.4 land and labour markets

Through the land reform instituted in 1975, all lands became public (state) property. Access to agricultural land became usufructuary. Each farm household is allotted a certain amount of land as a member of a PA. Renting out this holding, in part or

[11] Ox power is central to cultivation in the grain-plough complex of the central, eastern and northern parts of the country.

[12] For instance, in 1994/95 about 27 percent of total cultivated area was fertilized [CSA (1995)].

otherwise, was prohibited, and the prohibition was strictly enforced. In short there was no land market.

The rural land reform laws also banned the sale and purchase of labour. Partly due to problems of enforcement, the restriction was not fully effective. Indeed, there is some evidence indicating that transactions directly involving wage labour were wide-spread [Taffesse (1989), Taye (1992)]. For instance, for the sample households studied in Taffesse (1989), up to a quarter of the total farm labour is accounted for by hired labour. Such evidence suggests that there was a functioning rural labour market during the 1980's. Moreover, a variety of traditional labour exchange schemes are used by farm households.

As stated from the outset, our main objective is to examine the impact, on household choices, of compulsory grain delivery and rationing in manufactured consumer goods and fertilizers. The procedures described above allow us to analyse such effects on commodity and input demands, output supplies, and marketed surplus of farm households. In line with this, and in the interest of simplicity, we assume that a labour market not only exists but also clears. In this regard, there is some evidence, noted above, indicating that a labour market existed during the study period, and that farm households participated in that market. Strictly speaking, therefore, the substantive assumption made is that the labour market clears. The primary benefit of making that assumption is the removal of one possible source of non-separability of household production and consumption decisions.

2 The Model

Obviously Ethiopian peasant households make joint production and consumption decisions. Any analysis of their economic behaviour should allow for and study the nature of this property. Apart from providing additional insights concerning the functioning of semi-subsistence farming households, such analytical effort potentially leads to more accurate empirical results and policy implications [Singh, Squire, and Strauss (1986)]. It is, therefore, desirable to examine the impact of compulsory delivery and quantity constraints within the framework of agricultural household models. Towards that end the framework developed in Strauss (1986) and generalized in Mckay and Taffesse (1994) is adopted.

2.1 A first statement

In line with the discussion above, an Ethiopian farm household can be characterized as maximizing utility subject to a production function, a cash income (or explicit budget) constraint, and a time constraint. That discussion also suggests that, in the 1970's and 1980's, apart from these standard constraints, such a household may have faced quantity constraints in the form of ration levels. Below a model of an Ethiopian farm household reflecting these factors is constructed[13]. The main assumptions of the model are the following.

[13] Although the model is specifically applied to producers of annual crops in the 1980's, it can also be used to characterise the short-run behaviour of peasant farmers primarily producing other products. Obviously, for perennial crops planting decisions cannot be effectively handled and the output variables have to be redefined accordingly. It can also be suitably restricted to characterize the short-run behaviour of farm households after 1990.

1. There exists a single household utility function (U) - with household consumption of crops *(X_{ad} and X_{af})*, manufactured consumer goods *(X_{Mc} and X_{Mn})*, and leisure *(X_l)*, as its arguments - which is twice continuously differentiable, monotonically increasing and, quasiconcave.

2. The farm production function, $H(Q_{ad}, Q_{af}, L, V, A, K)$ - where Q_{ad}, Q_{af}, L, V, A, and K represent crop outputs, total labour input, fertilizer, total acreage, and non-land fixed input(s), respectively - is twice continuously differentiable and quasiconvex.

With the above assumptions, the farm household's static optimization problem can be stated as[14,15]:

$$\underset{X_{ad},X_{af},X_l,X_{Mc},X_{Mn},Q_{ad},Q_{af},L,F,V}{Maximise} \quad U(X_{ad}, X_{af}, X_l, X_{Mc}, X_{Mn}) \tag{2.1.1}$$

subject to:
cash income constraint[16]:

$$\begin{aligned} p_c^s R_{mc} + p_c^m (X_{Mc} - R_{Mc}) + p_n^s X_{Mn} - \{p_d^s \alpha Q_{ad} + p_d^m (Q_{ad} - \alpha Q_{ad} - X_{ad}) \\ + p_f^m (Q_{af} - X_{af}) + p_l^m (L - F) + p_v^s V\} \ \leq \ E \end{aligned} \tag{2.1.2}$$

time constraint:

$$X_l + F \leq T \tag{2.1.3}$$

production function:

$$H(Q_{ad},\, Q_{af},\, L, V, A, K) \leq 0 \tag{2.1.4}$$

ration levels:

$$X_{Mn} \leq R_{Mn} \tag{2.1.5}$$

$$V \leq R_v \tag{2.1.6}$$

where:

X_{ad} = own-consumption of the crop subject to CGD;

X_{af} = own-consumption of the crop free from CGD;

[14]The model applies to household decisions over a single agricultural cycle. Consequently, T, A and K are taken as fixed. Inventory decisions are ignored for the same reason.

[15]One of the major merits of static agricultural household models is that they allow the explicit modelling of joint production and consumption decisions. The introduction of risk in this framework appears to eliminate that possibility, unless time is also explicitly introduced. This is particularly significant in the present context since the impact of one consumption-side ration and one production-side ration on crop producers' choices constitute two main problems examined. The choice, therefore, is between a rather complicated dynamic model with risk, and a simple static model without risk. For the purpose at hand the latter model is deemed sufficient since that risk is not considered in what follows.

[16]After delivering the quota, the household in principle can make as much 'free' market purchases/sales of the crops subject to CGD as it desires. On the other hand EAMC did not supply grain to rural markets such that farm households have to purchase grain from the 'free' market means that the 'free' market price constitute the opportunity cost of own-consumption. This argument remains valid even in the presence of food aid. These facts justify evaluating X_{ad} at p_d^m.

X_l = leisure consumption;

X_{Mc} = consumption of 'convertible' rationed manufactured consumer goods;

X_{Mn} = consumption of 'non-convertible' rationed manufactured consumer goods;

R_{Mc} = the ration level of 'convertible' rationed manufactured consumer goods;

R_{Mn} = the ration level of 'nonconvertible' rationed manufactured consumer goods;

p_d^s = the procurement (or EAMC) price of the crop subject to CGD;

p_d^m = the 'free' or 'open' market price of the crop subject to CGD;

α = the proportion of the 'quota' out of the total output of the crop subject to CGD;

p_f^m = the market price of the crop free from CGD;

p_c^m = the (secondary) market price of 'convertible' rationed manufactured consumer goods;

p_c^s = the price at which the household purchases 'convertible' rationed manufactured consumer goods from the SC;

p_n^s = the price at which the household purchases 'nonconvertible' rationed manufactured consumer goods from the SC;

p_l^m = the market wage rate;

p_v^s = the price at which farmers purchase fertilizers form the SC;

Q_{ad} = the total output of the crop subject to CGD;

Q_{af} = the total output of the crop free from CGD;

L = total farm labour input;

F = family farm labour input;

$(L - F)$ = hired labour(hired-in if positive or hired-out if negative);

V = level of fertilizer use;

T = household's total time endowment;

E = non-wage, nonfarm net other income;

R_v = the ration level of fertilizers.

Recall that

$$p_d^s \alpha Q_{ad} + p_d^m (Q_{ad} - \alpha Q_{ad}) = p_d^m (1 - \rho) Q_{ad}$$

where, $\rho = [(1 - \frac{p_d^s}{p_d^m})\alpha]$, with $\rho \in [0, 1)$, represents the implicit output tax rate on the crop subject to CGD. The corresponding substitution is made in (2.1.2). Further noting that for appropriately defined leisure and/or time endowment variables the time constraint binds, we substitute for F again in (2.1.2). Rearranging we obtain a new budget constraint:

$$\begin{aligned}\overline{E} &= E + (p_c^m - p_c^s)R_{Mc} = \{[p_d^m X_{ad} + p_f^m X_{af} + p_l^m X_l + p_c^m X_{Mc} + p_n^s X_{Mn}] \\ &\qquad -[p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^s V] - p_l^m T\}\end{aligned} \tag{2.1.7}$$

where, $\overline{E}$ represents household exogenous income in the presence of 'convertible' rations R_{Mc}[17]. Being a combination of the cash income and time-endowment constraints, (2.1.7) is a full income budget constraint modified by the presence of 'convertible' rations.

Accordingly the optimization problem of the farm household can be restated as the maximization of (2.1.1) subject to constraints (2.1.4)-(2.1.7). The Lagrangian is formed in the usual way as:

$$\begin{aligned}\Phi_U &= U(X_{ad}, X_{af}, X_l, X_M, X_{Mn}) \\ &\quad +\lambda_E\{\overline{E} - [(p_d^m X_{ad} + p_f^m X_{af} + p_l^m X_l + p_c^m X_{Mc} + p_n^s X_{Mn}) \\ &\qquad -(p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^s V) - p_l^m T]\} \\ &\quad +\lambda_M(R_{Mn} - X_{Mn}) - \lambda_H H(Q_{ad}, Q_{af}, L, V, A, K) + \lambda_v(R_v - V)\end{aligned}$$

where; λ_H, λ_M, λ_v, and λ_E are Lagrangian multipliers related to constraints (2.1.4)-(2.1.7), respectively . In particular, λ_E represent the marginal utility of (full) income. Assuming interior solutions with binding constraints, the first-order conditions appear as[18]:

$$U_{ad} = \lambda_E p_d^m \tag{2.1.8}$$

$$U_{af} = \lambda_E p_f^m \tag{2.1.9}$$

$$U_l = \lambda_E p_l^m \tag{2.1.10}$$

$$U_{Mc} = \lambda_E p_c^m \tag{2.1.11}$$

$$U_{Mn} = \lambda_E[p_n^s + (\frac{\lambda_M}{\lambda_E})] \tag{2.1.12}$$

$$(p_d^m X_{ad} + p_f^m X_{af} + p_l^m X_l + p_c^m X_{Mc} + p_n^s X_{Mn}) - (p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^s V) - p_l^m T = \overline{E} \tag{2.1.13}$$

$$X_{Mn} = R_{Mn} \tag{2.1.14}$$

$$\frac{\lambda_H}{\lambda_E} H_{ad} = p_d^m(1-\rho) \tag{2.1.15}$$

[17] Note that in our model E (the standard exogenous income in consumer theory), p_c^m, p_c^s, and R_{Mc} are independent of household choice. Thus $\overline{E}$ is exogenous to the household. We refer to it as adjusted exogenous income.

[18] It is possible to use the Kuhn-Tucker approach and state first-order conditions accordingly. However, if farm households are assumed to be technically efficient (2.0.4) binds. If rationing constraints do not bind, then λ_M and/or λ_v reduce to zero and (2.0.12) and/or (2.0.14) drop out. Furthermore, the virtual-price approach subsequently adopted accommodates nonbinding constraints equally well. Details regarding the latter point can be found in Pitt and Lee (1986) and Pudney (1989). Note also that $p_n^* \equiv [p_n^s + (\frac{\lambda_M}{\lambda_E})]$ and $p_v^* \equiv [p_v^s + \frac{\lambda_v}{\lambda_E})]$ represent the virtual prices of X_{Mn} and V. These virtual prices will be characterized in detail below. As can be seen from (2.0.8)-(2.0.11) and (2.0.15)-(2.0.17) the virtual prices of unrationed goods coincide with their market prices.

$$\frac{\lambda_H}{\lambda_E} H_{af} = p_f^m \tag{2.1.16}$$

$$\frac{\lambda_H}{\lambda_E} H_l = -p_l^m \tag{2.1.17}$$

$$\frac{\lambda_H}{\lambda_E} H_v = -\left(p_v^s + \frac{\lambda_v}{\lambda_E}\right) \tag{2.1.18}$$

$$H(Q_{ad},\, Q_{af},\, L, V, A, K) = 0 \tag{2.1.19}$$

$$V = R_v \tag{2.1.20}$$

where: $U_i = \frac{\partial U}{\partial X_i}$, $X_i \in \{X_{ad}, X_{af}, X_l, X_{Mc}, X_{Mn}\}$; $H_l = \frac{\partial H}{\partial L}$; $H_v = \frac{\partial H}{\partial V}$; $H_{aj} = \frac{\partial H}{\partial Q_{aj}}$ $(j = d, f)$.

Before proceeding further we have to determine whether the model possesses the recursiveness property. Recursiveness in a farm household model implies that the circumstances which define the model allow the household to logically separate its temporally simultaneous production and consumption decisions. For this property to hold, the following conditions, relating to commodities both supplied and demanded by the household, have to be satisfied [Strauss (1986)][19]:

(a) the markets for all such goods exist and clear;

(b) all households are price-takers in all these markets; and

(c) such goods are homogenous[20].

The first two conditions are not violated by our model for two reasons. First, both rationed commodities are not simultaneously supplied and demanded by the household. X_{Mn} is a pure consumption good while V is a pure producer good. Second, it has been pointed out that, after delivering the 'quota', the household is commonly able to sell/purchase the crop(s) subject to CGD. We can further argue that they can do so as much as they desire at the local 'free' market prices. In short, as far as the short-run problem stated above is concerned, all the markets for goods both supplied and demanded by the farm household exist and clear. Combined with the assumption of homogenous factors and products this implies that our model is recursive[21]. More precisely, the farm household initially makes production decisions as a profit maximizer and, subject to the outcomes of that process, reach consumption decisions as a utility maximizer. A closer look at the set of first-order conditions reveals that this is the case. This set can be divided into two blocks. The first is the production block consisting of (2.1.15)-(2.1.20). No consumption-side choice variable enters this block, which implies that production decisions can be made independently of consumption decisions. In contrast, the consumption block formed by (2.1.8)-(2.1.14) is conditioned by production choices via the full income budget constraint.

[19] These are sufficient for a static model under certainty. Once time and risk are explicitly introduced, however, this type of recursiveness requires much stronger conditions including the existence of all relevant competitive state-contingent markets.

[20] With the common assumption of interior solutions (the level of all choice varibles are non-zero at the optimum), this condition can be relaxed in the sense that (a)-(b) are sufficient for the recursiveness property to hold [Strauss (1986)].

[21] Obviously, whether separability holds or not is ultimately an empirical question.

The solution to the farm household's optimization problem can be summarized by *rationed or restricted Marshallian demand functions* and the *rationed profit function*:

$$\begin{aligned}\widetilde{X}_i &= \widetilde{X}_i\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, R_{Mn}, \widetilde{\pi}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) + p_l^m T + \overline{E}\right)\\ &= \widetilde{X}_i\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, \overline{E}\right); \quad i = ad, af, l, Mc\end{aligned} \tag{2.1.21}$$

$$\widetilde{X}_{Mn} = \widetilde{X}_{Mn}\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, \overline{E}\right) = R_{Mn}. \tag{2.1.22}$$

$$\widetilde{\pi}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) = p_d^m(1-\rho)\widetilde{Q}_{ad} + p_f^m\widetilde{Q}_{af} - p_l^m\widetilde{L} - p_v^s\widetilde{V} \tag{2.1.23}$$

where, $\sim$ = 'rationed'[22]. The Marshallian demands (2.1.21) and (2.1.22) are dependent on production decisions through profits. They also depend on the ration levels R_{Mn} and R_v and that is why they are called rationed or restricted Marshallian demand functions. The rationed profit function $\widetilde{\pi}(.)$ shares all the properties of its unrationed counterpart[23]. It is linearly homogeneous, continuous, and convex in prices, increasing in output prices and decreasing in input prices. It is also twice-continuously differentiable since, by assumption, the production function has the same property. Moreover, applying Hotelling's lemma to $\widetilde{\pi}(.)$ results in the following output supply and input demand functions[24]:

$$\widetilde{Q}_{aj} = \widetilde{Q}_{aj}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right); j = d, f \tag{2.1.24}$$

$$\widetilde{L} = \widetilde{L}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) \tag{2.1.25}$$

$$\widetilde{V} = \widetilde{V}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) = R_v. \tag{2.1.26}$$

Because of their dependence on the ration level, R_v, these functions are characterized as *rationed or restricted output supply and variable-input demand functions*, respectively.

The above utility maximization problem can be reformulated as an expenditure (or cost of utility) minimization problem, i.e., the problem of choosing output, input, and consumption levels so as to minimize the expenditure necessary to achieve a given level of utility, $\overline{U}$. The two problems are identified as dual problems. That such a reformulation enables the derivation of more powerful results regarding consumer behaviour is well-known[25]. In addition it assumes a key role in the analysis of rationing

[22] Note that the full income of the farm household now consists of the farm profits (as conditioned by rationing), $\widetilde{\pi}$, the value of household time-endowment, $p_l^m T$, exogenous income, E, and the income/subsidy derived from 'convertible' rations, $(p_c^m - p_c^s)R_{Mc}$. As a result it can be referred to as *rationed adjusted full income*, $\widetilde{Y}_F'$. Symbolically:

$$\widetilde{Y}_F' = \widetilde{\pi}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) + p_l^m T + E + (p_c^m - p_c^s)R_{Mc}$$

[23] For a general discussion of the profit function see MacFadden and Fuss (1978), and Chambers (1988).

[24] Recall that:

$$p_d^m(1-\rho) = p_d$$

The average price defined this way represents the household's marginal value (or net price) of a unit of Q_{ad}. In applying Hotelling's lemma, we have used p_d directly. This holds for all cases of partial differentiation of the profit function with respect to the price of Q_{ad}.

[25] For further details see, among others, Deaton and Muellbauer (1980), and Cornes (1992).

below. In the present context, the standard formulation of the expenditure minimization problem has to be modified. First, to accommodate the producer-consumer nature of the farm household, it is necessary to minimize net, rather than total, expenditure, with incomes (or revenues) defined as negative expenditures. Thus, E has to be used. Second, to account for the presence of 'convertible' rations, adjusted exogenous income, $\overline{E}$, has to be used[26].

Formally the restatement takes the following form.

$$\underset{X_{ad},X_{af},X_l,X_{Mc},X_{Mn},Q_{ad},Q_{af},L,F,V}{Minimize} \{p_c^m X_{Mc} + p_n^s X_{Mn} + p_l^m (L - F) + p_v^s V \\ -p_d^m[(1-\rho)Q_{ad} - X_{ad}] + p_f^m(Q_{af} - X_{af})\} \tag{2.1.27}$$

subject to:

$$U(X_{ad}, X_{af}, X_l, X_M, X_{Mn}) \geq \overline{U} \tag{2.1.28}$$

$$H(Q_{ad}, Q_{af}, L, V, A, K) \leq 0 \tag{2.1.29}$$

$$X_l + F \leq T \tag{2.1.30}$$

$$X_{Mn} \leq R_{Mn} \tag{2.1.31}$$

$$V \leq R_v \tag{2.1.32}$$

where all the variables are as defined above. After substituting for F in (2.1.27) from (2.1.30) and rearranging, we can write the Lagrangian as[27]:

$$\begin{aligned} \Phi_E = & -\{[p_d^m X_{ad} + p_f^m X_{af} + p_l^m X_l + p_c^m X_{Mc} + p_n^s X_{Mn}] \\ & -[p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^s V] - p_l^m T\} \\ & +\mu_U\left(U(X_{ad}, X_{af}, X_l, X_{Mc}, X_{Mn}) - \overline{U}\right) - \mu_H H(Q_{ad}, Q_{af}, L, V, A, K) \\ & +\mu_M(R_{Mn} - X_{Mn}) + \mu_v(R_v - V) \end{aligned}$$

With binding constraints and interior solutions the corresponding first-order (necessary) conditions are[28]:

$$U_{ad} = \left(\frac{1}{\mu_U}\right) p_d^m \tag{2.1.8'}$$

$$U_{af} = \left(\frac{1}{\mu_U}\right) p_f^m \tag{2.1.9'}$$

$$U_l = \left(\frac{1}{\mu_U}\right) p_l^m \tag{2.1.10'}$$

$$U_{Mc} = \left(\frac{1}{\mu_U}\right) p_c^m \tag{2.1.11'}$$

[26]As noted above $\overline{E}$ represents household exogenous income adjusted by the (exogenous) income/subsidy from 'convertible' rations R_{Mc}. That is why the objective function takes the from of (2.1.27).

[27]In stating the Lagrangean we have used the fact that minimising a function is equivalent to maximising the negative of that function.

[28]Compare (2.1.12') with (2.1.12) under utility maximisation. Anticipating subsequent definitions we can characterize $\left(p_n^s + \frac{\lambda_M}{\lambda_E}\right)$ and $(p_n^s + \mu_M)$ as the uncompensated and the compensated virtual prices of X_{Mn}, respectively.

$$U_{Mn} = \left(\frac{1}{\mu_U}\right)[p_n^s + \mu_M] \qquad (2.1.12')$$

$$U(X_{ad}, X_{af}, X_l, X_{mc}, X_{Mn}) = \overline{U} \qquad (2.1.33)$$

$$X_{Mn} = R_{Mn} \qquad (2.1.14')$$

$$\mu_H H_{ad} = p_d^m(1-\rho) \qquad (2.1.15')$$

$$\mu_H H_{af} = p_f^m \qquad (2.1.16')$$

$$\mu_H H_l = -p_l^m \qquad (2.1.17')$$

$$\mu_H H_v = -(p_v^s + \mu_v) \qquad (2.1.18')$$

$$H(Q_{ad},\, Q_{af},\, L, V, A, K) = 0 \qquad (2.1.19')$$

$$V = R_v \qquad (2.1.20')$$

The solution to this problem can be summarised by the *rationed or restricted (minimum) net expenditure function:*

$$\begin{aligned}\widetilde{e}'(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, \overline{U}) &= (p_d^m\widetilde{X}_{ad}^h + p_f^m\widetilde{X}_{af}^h + p_l^m\widetilde{X}_l^h + p_c^m\widetilde{X}_{Mc}^h + p_n^s\widetilde{X}_{Mn}^h) \\ &\quad -[p_d^m(1-\rho)\widetilde{Q}_{ad} + p_f^m\widetilde{Q}_{af} - p_l^m\widetilde{L} - p_v^s\widetilde{V}] - p_l^m T\end{aligned} \qquad (2.1.34)$$

where $\widetilde{e}'$ is the minimum net expenditure required to attain the given utility level $\overline{U}$. The minimum net expenditure function $\widetilde{e}'(.)$ is concave in prices, increasing in prices and utility, and decreasing and convex in R_{Mn} and R_v[29]. By the corresponding property of the utility function it is also twice-continuously differentiable. Note that minimized net expenditure is composed of minimized total expenditure on consumption goods, maximized farm profits, and the value of household time-endowment. Hence, it is possible rewrite the net expenditure function as:

$$\widetilde{e}'(.) = \widetilde{e}\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, R_{Mn}, \overline{U}\right) + \widetilde{\pi}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) + p_l^m T \qquad (2.1.35)$$

where $\widetilde{e}(.) = p_d^m\widetilde{X}_{ad}^h + p_f^m\widetilde{X}_{af}^h + p_l^m\widetilde{X}_l^h + p_c^m\widetilde{X}_{Mc}^h + p_n^s\widetilde{X}_{Mn}^h$, and $\widetilde{\pi}(.)$ is as defined above. $\widetilde{e}(.)$ can be characterised as the *rationed or restricted (minimum) total expenditure function.* It shares all the properties of the net expenditure function. Further, by Shepherd's lemma, the first-order partial differentiation of $\widetilde{e}'(.)$ with respect to prices generates the Hicksian net demands, while doing the same to $\widetilde{e}(.)$ results in rationed Hicksian total demands[30,31]. In addition to prices and the given utility level, these demands depend on the ration level R_{Mn} and/or R_v. Consequently they are referred to as *rationed or restricted Hicksian (or compensated) net and total demand functions*, respectively.

Given separable consumption and production decisions, production choices are independent of $\overline{U}$. Thus, only Hicksian total demands for consumption goods assume a different form. The latter are expressed as:

$$\widetilde{X}_i^h = \widetilde{X}_i^h\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, R_{Mn}, \overline{U}\right); \quad i = ad, af, l, Mc \qquad (2.1.36)$$

[29] Deaton (1981) establishes these properties of the expenditure function in the pure consumer case. Using analogous arguments it is possible to show that these properties hold in the producer-consumer case.

[30] Note that in this context supplies are defined as negative demands.

[31] Further details concerning $\widetilde{e}(.)$ are provided in Neary and Roberts (1980), Deaton (1981), and Cornes (1992).

$$\widetilde{X}^h_{Mn} = \widetilde{X}^h_{Mn}\left(p^m_d, p^m_f, p^m_l, p^m_c, p^s_n, R_{Mn}, \overline{U}\right) = R_{Mn} \tag{2.1.37}$$

Finally note that the rationed equilibrium of the farm household can now be expressed as the point at which rationed minimum net expenditure is equated with adjusted exogenous income:

$$\widetilde{e'}(p^m_d, p^m_f, p^m_l, p^m_c, p^s_n, p^s_v, \rho, R_{Mn}, R_v, T, A, K, \overline{U}) = \overline{E} \tag{2.1.38}$$

At this point, the well-known procedure of substituting minimized expenditure for income in uncompensated demands can be used to demonstrate the equality of Marshallian and Hicksian demands[32]:

$$\widetilde{X}_i\left[p^m_d, p^m_f, p^m_l, p^m_c, p^s_n, p^s_v, \rho, R_{Mn}, R_v, T, A, K, \widetilde{e'}(.)\right] = \widetilde{X}^h_i\left(p^m_d, p^m_f, p^m_l, p^m_c, p^s_n, R_{Mn}, \overline{U}\right) \tag{2.1.39}$$

where $i \in (ad, af, l, Mc, Mn)$. Following Cook (1972) equation (2.1.39) is also used to derive the Slutsky equation.

2.2 A virtual-price restatement

The model and results presented above allow us to analyse the behaviour of farm households under the stipulated rationed environment. Nevertheless, recasting the problem in the virtual-price framework affords the possibility of obtaining more general results. The principal advantage of doing so lies in the possibility of deriving matched rationed and unrationed demand and supply equations, so as to exploit the known global properties of the latter in studying the former. Towards that end, we proceed as follows. First we define the virtual prices of V and X_{Mn}. Second we summarize the results of unrationed utility maximization and expenditure minimization at virtual prices. Finally, the relations between rationed and unrationed demand as well as expenditure functions are stated.

The virtual price of a rationed good is the price at which a rationed economic agent (in the present case a farm household) will voluntarily choose the ration level [Neary and Roberts (1980)][33]. Such a price is endogenous to the household in that it is determined, not only by exogenous variables (including market prices and exogenous ration levels), but also by household preferences, endowments, and production technology. In the present case there are two rationed goods, V and X_{Mn}. In the case of V, a pure production good, the recursiveness of our model means that the endogeneity of the virtual price stems from its dependence on household resource endowments (specifically, A and K) and technology of production[34]. We denote this price by p^*_v. In contrast, the virtual price of X_{Mn} is defined either at a given level of exogenous income or a given level of utility[35]. Accordingly, this price assumes

[32] This procedure is known as Cook's trick after Cook (1972) who first suggested it.

[33] Regarding the essential features of the virtual price approach, including the conditions for the existence and uniqueness of virtual prices, as well as its application to consumer behaviour under rationing see Neary and Roberts (1980), Deaton (1981), and Neary (1987). In the agricultural household modelling literature, Strauss (1986) seems to be the first to explicitly employ these prices. See also McKay and Taffesse (1994, 1995) for a detailed examination.

[34] In other words, it does not depend on household income or utility level. Hence, it assumes only one form.

[35] A virtual price is essentially an inverse demand function. As such it assumes uncompensated and compensated forms. This distinction is only implicit in Neary and Roberts (1980). Again consult Strauss (1986); and McKay and Taffesse (1994) for a detailed examination of the properties of these prices and their application in agricultural household models.

two forms. They are, respectively, the uncompensated virtual price (p_n^*) and the compensated virtual price $\left(p_n^{*h}\right)$.

Recall the restricted demands for and supplies of the unrationed goods (2.1.21) and (2.1.24)-(2.1.26)[36]. These demands and supplies are defined at $\overline{E}$. Given prices and the exogenously fixed R_{Mn} and R_v, however, it is as if the household is subject to a budget constraint of $\left(\overline{E} - p_n^s R_{Mn} - p_v^s R_v\right)$. Suppose that, with other prices remaining the same, p_n^s and p_v^s change by dp_n^s and dp_v^s, respectively. Such a change can only produce an income effect on the demand for X_i $(i = ad, af, l, Mc)$ by reducing what the household is left with for spending on them. However, if, at the same time, the household's income is adjusted by an amount equal to $(dp_n^s R_{Mn} + dp_v^s R_v)$, that income effect is neutralized.. In other words, quantities demanded and supplied and the corresponding utility level at $\left[(p_n^s + dp_n^s), (p_v^s + dp_v^s), (\overline{E} + dp_n^s R_{Mn} + dp_v^s R_v)\right]$ are the same as those at $\left[p_n^s, p_v^s, \overline{E}\right]$[37]. A precisely analogous process is involved in defining the uncompensated virtual price of X_{Mn} and V except, of course, that such a price is endogenous to the household. The objective is to determine the price-income configuration at which the household voluntarily chooses the rationed production-consumption point. Towards that end, change the price of X_{Mn} and V to p_n^* and p_v^*, respectively, and simultaneously compensate the household for the imposition of the ration. The compensation, being the amount of income adjustment necessary to keep the household at the rationed production-consumption point given the new price set, is equal to $(p_n^* - p_n^s)\, R_{Mn} + (p_v^* - p_v^s)\, R_v$. Consequently the argument made earlier in this paragraph applies such that demands and supplies are re-evaluated at $\left[p_n^*, p_v^*, \left(\overline{E} + (p_n^* - p_n^s)R_{Mn} + (p_v^* - p_v^s)R_v\right)\right]$. In the spirit of the usage of the term "virtual income" by Neary (1987) for the pure consumer case we may refer to the sum:

$$\left(\overline{E} + (p_v^* - p_v^s)R_v + (p_n^* - p_n^s)R_{Mn}\right) = \overline{E}' \tag{2.2.1}$$

as *virtual adjusted exogenous income.*

Accordingly, we restate the primal and dual optimization problems of the farm household in terms of virtual prices, virtual adjusted exogenous income. In so doing we have to drop the rationing constraints generated by 'nonconvertible' rations. We start with utility maximization. This problem now involves maximising (2.1.1) subject to (2.1.4), re-evaluated at virtual prices, virtual adjusted exogenous income, and (2.1.7). The Lagrangian of the problem can be stated as:

$$\begin{aligned}
\Phi_U^* \;=\; & U(X_{ad}, X_{af}, X_l, X_M, X_{Mn}) \\
& + \lambda_E^* \{ \left(\overline{E} + (p_v^* - p_v^s)R_v + (p_n^* - p_n^s)R_{Mn}\right) \\
& \qquad - [(p_d^m X_{ad} + p_f^m X_{af} + p_l^m X_l + p_c^m X_{Mc} + p_n^* X_{Mn}) \\
& \qquad - (p_d^m (1-\rho) Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^* V) - p_l^m T]\} \\
& - \lambda_H^* H(Q_{ad},\, Q_{af},\, L, V, A, K)
\end{aligned}$$

[36] The manner in which the terms 'rationed' and 'unrationed' are used may be slightly confusing. 'Rationed goods' identify those goods which are directly subject to a quantity constraint, while 'unrationed goods' are those which are free from such a constraint. 'Rationed demands and supplies', on the other hand, refer to demands and supplies that are directly conditioned by the ration levels of quantity constrained goods, whereas 'unrationed demands and supplies' are not conditioned in this particular way. .

[37] Cornes (1992, p. 173) uses essentially the same argument to make a related but different point.

where λ_E^* and λ_H^* are Lagrangian multipliers.

Unrationed Marshallian demands (at virtual prices and virtual adjusted exogenous income) and the *unrationed profit function* (at virtual prices) summarize the resultant solution as:

$$\begin{aligned} X_i &= X_i\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, \overline{E}' + \pi\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) + p_l^m T\right) \\ &= X_i\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, \overline{E}'\right) \end{aligned} \tag{2.2.2}$$

$$\pi = \pi\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) = p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^* V. \tag{2.2.3}$$

where $i = ad, af, l, Mc, Mn$. As usual the first-order price derivatives of $\pi(.)$ are, by Hotelling's lemma, the respective *unrationed or unrestricted output supply and variable-input demand functions.* These demands and supplies are now evaluated at virtual prices:

$$Q_{aj} = Q_{aj}\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) \tag{2.2.4}$$

$$L = L\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) \tag{2.2.5}$$

$$V = V\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) \tag{2.2.6}$$

where $j = d, f$.

The restatement allows the matching of rationed and unrationed demands and supplies such that the global properties of the former can be examined in terms of those of the latter. This is an advantage because the well known features of unrationed demands and supplies can be exploited in the process. In the postulated context of exogenous ration levels and excess demand for X_{Mn} and V,

$$p_n^* \geq p_n^s \text{ and } p_v^* \geq p_v^s$$

Thus, the change from (p_n^s, p_v^s) to (p_n^*, p_v^*) is equivalent to a rise in the prices of X_{Mn} and V. Given exogenously determined levels of R_{Mn} and R_v, and other variables remaining the same, such a rise increases the expenditure of the household on the rations. However it does not affect the optimal levels of outputs and inputs provided that the household is compensated for this loss in income. Therefore, at the rationed production point the rationed demands and supplies are equal to their unrationed counterparts so long as the latter are evaluated at virtual prices (and virtual income in the case of commodity demands):

$$\widetilde{X}_i(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, \overline{E}) = X_i[p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, \overline{E}'] \tag{2.2.7}$$

$$\widetilde{Q}_{aj}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) = Q_{aj}\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) \tag{2.2.8}$$

$$\widetilde{L}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) = L\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) \tag{2.2.9}$$

$$\widetilde{V}\left(p_d^m, p_f^m, p_l^m, p_v^s, \rho, R_v, A, K\right) = V\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) \tag{2.2.10}$$

where $i = ad, af, l, Mc, Mn$ and $j = d, f$.

In this regard, the characterization of a virtual price implies that,

$$V\left(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K\right) = R_v$$

$$X_{Mn}\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, \overline{E}'\right) = R_{Mn}$$

which implicitly define the virtual prices of V and X_{Mn} as:

$$p_v^* = p_v^*\left(p_d^m, p_f^m, p_l^m, \rho, R_v, A, K\right). \tag{2.2.11}$$

$$p_n^* = p_n^*\left(p_d^m, p_f^m, p_l^m, p_c^m, \rho, R_v, R_{Mn}, T, A, K, \overline{E}'\right) \tag{2.2.12}$$

Note that p_n^* represents the household's uncompensated virtual price of X_{Mn}. In other words, it is the price at which the unrestricted uncompensated demand for X_{Mn} equals the ration level R_{Mn}. This price is obviously endogenous to the household in the sense that, together with the relevant exogenous variables, household preferences and endowments determine its level. In contrast, recall that p_n^* assumes only one form.

The expenditure minimization problem can be restated in an analogous fashion. The problem consists of minimising (2.1.27) at (compensated) virtual prices subject to (2.1.28) and (2.1.29). The corresponding Lagrangian can be written as:

$$\begin{aligned}\Phi_E^* \quad = \quad & -\{[p_d^m X_{ad} + p_f^m X_{af} + p_l^m X_l + p_c^m X_{Mc} + p_n^{*h} X_{Mn}] \\ & -[p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^* V] - p_l^m T\} \\ & +\mu_U^*\left(U(X_{ad}, X_{af}, X_l, X_{Mc}, X_{Mn}) - \overline{U}\right) \\ & -\mu_H^* H(Q_{ad}, Q_{af}, L, V, A, K)\end{aligned}$$

where μ_U^* and μ_H^* are Lagrangian multipliers. The solution to this minimisation problem is the *unrationed or unrestricted (minimum) net expenditure function*:

$$\begin{aligned}e'(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, \overline{U}) \quad = \quad & (p_d^m X_{ad}^h + p_f^m X_{af}^h + p_l^m X_l^h + p_c^m X_{Mc}^h + p_n^{*h} X_{Mn}^h) \\ & -[p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^* V] - p_l^m T\end{aligned} \tag{2.2.13}$$

As is obvious from (2.1.13), $e'(.)$ can also be rewritten as:

$$e'(.) = e(p_d^m, p_f^m, p_l^m, p_c^m, p_n^{*h}, \overline{U}) - \pi(p_d^m, p_f^m, p_l^m, p_v^*, \rho, A, K) - p_l^m T$$

where:

$$\begin{aligned}e(.) \quad &= \quad p_d^m X_{ad}^h + p_f^m X_{af}^h + p_l^m X_l^h + p_c^m X_{Mc}^h + p_n^{*h} X_{Mn}^h \\ \pi(.) \quad &= \quad p_d^m(1-\rho)Q_{ad} + p_f^m Q_{af} - p_l^m L - p_v^* V\end{aligned}$$

The function $e(.)$ constitutes the *unrationed or unrestricted (minimum) total expenditure function.*

Both $e'(.)$ and $e(.)$ possesses the concavity and derivative properties of minimum expenditure functions. Specifically, Shephard's lemma implies that the first-order partial derivatives of $e'(.)$ and $e(.)$ with respect to prices are *unrationed (or unrestricted) Hicksian net and total demands* at virtual prices, respectively.

>From the above restatement of the household's optimization problem a number of results are obtained.

1) Since both are evaluated at the same given utility level $\overline{U}$, unrestricted Hicksian total demands at virtual prices are equal to their rationed counterparts:

$$\widetilde{X}_i^h(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, R_{Mn}, \overline{U}) = X_i^h(p_d^m, p_f^m, p_l^m, p_c^m, p_n^{*h}, \overline{U}) \tag{2.2.14}$$

where $i = ad, af, l, Mc, Mn$.

2) That production decisions are independent of consumption choices imply that the unrationed profit function, and thus, the unrationed output supply and input demand functions retain the form they have under utility maximization.. These are stated as (2.2.3)-(2.2.6) above.

3) The equality, at the rationed consumption point, between the unrationed demand for X_{Mn} and the ration level:

$$X^h_{Mn}(p^m_d, p^m_f, p^m_l, p^m_c, p^{*h}_n, \overline{U}) = R_{Mn}$$

implicitly defines p^{*h}_n (the compensated virtual price) as:

$$p^{*h}_n = p^{*h}_n(p^m_d, p^m_f, p^m_l, p^m_c, R_{Mn}, \overline{U}) \tag{2.2.15}$$

It is the price which equates unrationed Hicksian total demand for X_{Mn} with the ration level. Being a virtual price p^{*h}_n is also endogenous to the household.

4) A comparison between (2.1.35) and (2.2.13) provides us with the relationship between the rationed and unrationed net and total expenditure functions:

$$\widetilde{e}'(.) = e'(.) - (p^*_v - p^s_v)R_v - (p^{*h}_n - p^s_n)R_{Mn} \tag{2.2.16}$$

$$\widetilde{e}(.) = e(.) - (p^*_v - p^s_v)R_v - (p^{*h}_n - p^s_n)R_{Mn} \tag{2.2.17}$$

5) Finally, the unrationed equilibrium of the farm household is characterised by[38]:

$$e'(.) = \overline{E} + (p^*_v - p^s_v)R_v + (p^{*h}_n - p^s_n)R_{Mn} = \overline{E}' \tag{2.2.18}$$

At this point unrationed Marshallian and Hicksian total demands are equal:

$$X_i\left(p^m_d, p^m_f, p^m_l, p^m_c, p^*_n, p^*_v, \rho, T, A, K, e'(.)\right) = X^h_i(p^m_d, p^m_f, p^m_l, p^m_c, p^{*h}_n, \overline{U}) \tag{2.2.19}$$

where $i = ad, af, l, Mc, Mn$. The Slutsky relation based on this equality thus applies. Furthermore, (2.2.19) implies that the uncompensated virtual price (p^*_n) and the compensated virtual price $\left(p^{*h}_n\right)$ are equal in magnitude at equilibrium, but are affected differently by exogenous variables. Hence we have:

$$p^*_n\left(p^m_d, p^m_f, p^m_l, p^m_c, \rho, R_v, R_{Mn}, T, A, K, e'(.)\right) = p^{*h}_n(p^m_d, p^m_f, p^m_l, p^m_c, R_{Mn}, \overline{U} \tag{2.2.20}$$

We now have derived all the major results necessary to proceed to comparative static analysis.

3 A note on allocative efficiency

Consider the first-order conditions of the utility maximization problem of the farm household[39]. Under the stipulated circumstances production decisions do not depend

[38] Rationed equilibrium requires that $\overline{E} = \widetilde{e}'$. Consequently, unrationed equilibrium at the same production-consumption point has to satisfy this condition, albeit at virtual prices. From equation (2.2.16) we obtain:

$$e'(.) = \widetilde{e}'(.) + (p^*_v - p^s_v)R_v + (p^{*h}_n - p^s_n)R_{Mn}$$

Since, $\widetilde{e}' = \overline{E}$, we get:

$$e'(.) = \overline{E} + (p^*_v - p^s_v)R_v + (p^{*h}_n - p^s_n)R_{Mn} = \overline{E}'$$

[39] The result concerning allocative efficiency obtained below does not depend on our choice of the specific form of the household's optimization problem.

on consumption decisions. Thus (2.2.15)-(2.1.20) define the production equilibrium of the household as conditioned by CGD and rationing in V. In this regard, a comparison with the production equilibrium which would obtain in the absence of CGD reveals the potential impact of CGD on household allocative efficiency.

The first-order conditions can be restated in a more conventional form by applying the Implicit Function Theorem. For our purpose, the most relevant of these relate to Q_{ad}, and appear as[40]:

$$(1-\rho)p_d^m MP_i^{ad} = p_i \tag{3.1}$$

where: $MP_i^{ad} = -\left(\frac{H_i}{H_{ad}}\right)$, which, by the Implicit Function Theorem, expresses the marginal product of variable input i ($i = l, v$) in the production of Q_{ad}. The conditions also apply in the absence of the 'quota' system (i.e., with $\rho = 0$) and the corresponding market price of Q_{ad}. Explicitly, with the latter price denoted by p_d^e :

$$p_d^e MP_i^{ad} = p_i \tag{3.2}$$

Equation (3.1) implies that the 'quota' system induces households to produce Q_{ad} at a point at which only a fraction of the marginal value product (at p_d^m) of input i is equated with that input's price. Given the assumptions about the production function, the output thus produced is lower than what would have been produced at the point of equality between the entire marginal value product (at p_d^m) and the corresponding input price. On the other hand, with (3.2), the whole of the marginal value product (at p_d^e) is equated with p_i. Suppose the following inequalities hold between the prices of Q_{ad} :

$$(1-\rho)p_d^m = p_d < p_d^e \leq p_d^m.$$

Then, given other prices and fixed input levels, the introduction of the 'quota' system forces farm households to produce less of Q_{ad} compared to their pre-'quota' operations. Formally, for a given p_i :

$$p_d MP_i^{ad}\|_{I_{ad}} = p_d^e MP_i^{ad}\|_{I_{ad}^e} \tag{3.3}$$

where I_{ad} and I_{ad}^e represent the (L, V) allocations to Q_{ad} production at (p_d, p_i) and (p_d^e, p_i), respectively. Since $p_d < p_d^e$, the equality in (3.3) holds only if MP_i^{ad} at I_{ad} is greater than the corresponding marginal product at I_{ad}^e. Under the assumed technical efficiency of farm households, the latter implies that I_{ad} is less than I_{ad}^e[41]. In other words, compared to what it would have done at (p_d^e, p_i), the household allocates less L and V to Q_{ad} at (p_d, p_i).

Therefore, by depressing the return on the production of Q_{ad} (or, equivalently, raising the relative price of inputs in terms of Q_{ad} - $\frac{p_i}{(1-\rho)p_d^m}$ is greater than $\frac{p_i}{p_d^e}$), the 'quota' system eventuates changes in the mix of crops cultivated by farm households. The strength and significance of the disincentive thus created increases with the share of Q_{ad} in total crop output. Indeed, if Q_{ad} is the only crop produced by the

[40]The expression that applies to the marginal rate of transformation between Q_{ad} and Q_{af} ($\mathrm{MRT}_{d,f}$) is:

$$MRT_{d,f} = -\left(\frac{H_{af}}{H_{ad}}\right) = -\left(\frac{p_f^m}{(1-\rho)p_d^m}\right).$$

[41]More specifically this result requires that, given A and K, the production technology displays (at least locally) decreasing returns to scale with respect to L and V.

farm household, this disincentive leads to reductions in the level of overall input use, thereby making it unable to fully realize the existing profit-generating potential. Consequently, in this particular case, the farm household becomes allocatively less efficient in production[42,43]. The revenue-loss also rises with α, and decreases with p_d^s and p_d^m. Evidently it does not arise if p_d is equal to p_d^e. In fact, an average price in excess of p_d^e may, within the bounds of production technology and levels of fixed inputs, encourage expansion in the production of Q_{ad}. Therefore, considering the 'quota' as a proportional output tax enabled us to formalize, in a very simple manner, the often-made argument that forced grain procurement may lead to an inefficient outcome in farmers' productive activity[44]. The model suggests that whether this inefficiency occurs depends on crop substitution possibilities in production and the direction and magnitude of the deviation of p_d from p_d^e.

4 Comparative Statics

The model set out above contains a relatively large number of exogenous as well as endogenous variables, including two virtual prices. It is neither necessary nor interesting to consider the impact and/or the response of all of them explicitly. Instead the comparative static properties of the model are examined in two parts. The first part, presented in Appendix I, identifies the ways in which an exogenous variable impacts on variables endogenous to the model. It involves the derivation of expressions for the effect of a general exogenous variable on virtual prices, household welfare, output supplies, and commodity demands. The specific applications of the general expressions thus obtained comprise the second part, which is contained in this section. This part focuses on the influence of CGD and rationing on household behaviour. Specifically considered are the comparative static effects of α, R_{Mn}, and p_d^m on output supplies, commodity demands, marketed surplus, and household welfare.

4.1 Changes in the 'quota' level

Consider a change in the proportion of output that a farm household has to sell to the government. In the model set out above this change impacts on both production and consumption, and through them, on marketed surplus. It also affects household welfare as a consequence.

[42] This result is analogous to tenants' inefficiency in the 'Marshallian' model of sharecropping. In both cases part of the farmer's produce accrues, albeit in different forms, to another agent. However, the present model addresses a different problem within a different institutional setup and, partly because of that, does not suffer from the well-known shortcomings of the 'Marshallian' model. For instance, the problem of infinite demand for land does not arise because the land tenure system makes household land-holding exogenous. The critique appealing to the voluntariness of tenant-landlord contracts and the implied gains from trade does not apply because procurement is forced. See Quibria and Rashid (1984) for a review of the literature on sharecropping.

[43] Evidently the disincentive effect is likely to assume greater importance to long-term decisions subject to uncertainty, including farm investment and innovation.

[44] Schiff and Valdes (1994) argue that excessive direct and indirect taxation of agriculture negatively affects its performance, and provide supporting evidence from eighteen countries. Schiff (1993), and in the Ethiopian case Franzel et al (1989) forward the same argument. However, none of them provide a formal microeconomic model of this particular effect.

4.1.1 output responses

In our model a change in α generates output effects because it affects the average price p_d and thus, the profitability of a crop enterprise subject to the 'quota'[45]. The argument then follows that farm households, as profit-maximizers, incorporate the change in their production decisions. Starting with the impact on Q_{ad}, we accordingly apply (AI.13) for $j = d$ and $\tau = \alpha$, to obtain:

$$\frac{\partial \widetilde{Q}_{ad}}{\partial \alpha} = \pi_{d,\alpha} - \pi_{d,v}\left(\pi_{v,v}\right)^{-1}\pi_{v,\alpha}$$

where: $\pi_{d,\alpha} = \frac{\partial^2 \pi}{\partial \alpha \partial pd}$, $\pi_{d,v} = \frac{\partial^2 \pi}{\partial p_v^* \partial p_d}$, $\pi_{v,v} = \frac{\partial^2 \pi}{\partial p_v^* \partial p_v^*}$, and $\pi_{v,\alpha} = \frac{\partial^2 \pi}{\partial \alpha \partial p_v^*}$. The two terms on the right-hand-side are, respectively, the direct and virtual-price effects. That α operates through p_d means, $\pi_{k,\alpha} = \pi_{k,d}\frac{\partial p_d}{\partial \alpha}$, $(k = ad, af, v, l)$, while the definition of p_d implies $\frac{\partial p_d}{\partial \alpha} = (p_d^s - p_d^m)$. By Young's theorem, which applies to the twice-continuously differentiable profit function, cross-price effects are equal, i.e., $\pi_{j,k} = \pi_{k,j}$. In the present case this results $\pi_{d,v} = \pi_{v,d}$. Subsequent substitution and rearrangement leads to:

$$\frac{\partial \widetilde{Q}_{ad}}{\partial \alpha} = (p_d^s - p_d^m)(\pi_{d,d}) - (p_d^s - p_d^m)\left(\pi_{d,v}\right)^2\left(\pi_{v,v}\right)^{-1} \tag{4.1}$$

With $p_d^s < p_d^m$, $(p_d^s - p_d^m)$ is negative. On the other hand, by the convexity of the profit function, $\pi_{d,d}$, $\pi_{v,v}$, and thus $(\pi_{v,v})^{-1}$, are positive. Since $(\pi_{d,v})^2$ is also positive, (4.1) shows that the direct and the virtual-price effects work in opposite directions. Moreover, the circumstances of a farm household determine whether or not all of these effects operate simultaneously. In this regard (4.1) accommodates the following possibilities. First consider the impact of a rise in α on the level of Q_{ad} produced by a farm household which is subject to the 'quota' and a binding ration in V. The rise in α translates into a fall in p_d and engenders a reduction in Q_{ad} - the direct effect is negative. In contrast, the virtual-price effect is positive if it is assumed that V is a normal input $(\pi_{v,v} > 0)$. A fall in p_d reduces the demand for V, thereby lowering its virtual price at the given level of R_v. The decline in p_v^*, in turn, exerts a positive influence on output by lowering marginal cost. The net effect however, is negative. To see this, multiply (4.1) by $(\pi_{v,v})(\pi_{v,v})^{-1}$, and collect terms to get:

$$\frac{\partial \widetilde{Q}_{ad}}{\partial \alpha} = (p_d^s - p_d^m)\left[(\pi_{d,d})(\pi_{v,v}) - (\pi_{d,v})^2\right]\left(\pi_{v,v}\right)^{-1}$$

[45] As noted earlier, we use the following results in what follows.

$$\frac{\partial \pi}{\partial z} = \frac{\partial \pi}{\partial p_d}\frac{\partial p_d}{\partial z} = \frac{\partial \pi}{\partial p_d}\frac{\partial[(1-\rho)p_d^m]}{\partial z}$$

$$\frac{\partial[(1-\rho)p_d^m]}{\partial z} = \frac{\partial p_d^m}{\partial z}(1-\rho) + \frac{\partial(1-\rho)}{\partial z}p_d^m = \frac{\partial p_d^m}{\partial z}(1-\rho) - \frac{\partial \rho}{\partial z}p_d^m$$

$$\frac{\partial \rho}{\partial z} = \frac{\partial}{\partial z}\left[\left(1 - \frac{p_d^s}{p_d^m}\right)\alpha\right]$$

where $z \in \{p_d^m, p_d^s, \alpha\}$. Specifically:

$$\frac{\partial p_d}{\partial \alpha} = (p_d^s - p_d^m); \qquad \frac{\partial p_d}{\partial p_d^s} = \alpha; \qquad \frac{\partial p_d}{\partial p_d^m} = (1-\alpha)$$

Since it is a 2×2 principal minor of the Hessian of the convex profit function, the term in square brackets is positive[46]. As a result the whole expression is negative. It implies that, as far as the net impact on Q_{ad} is concerned, the direct effect of α dominates its virtual-price effect. Second consider a household for which the rationing constraint does not bind. In that case the entire output response to the rise in α is captured by the first term of (4.1), which is negative. Third, with or without a binding ration in V, the extent to which Q_{ad} falls increases with the size of the price differential $(p_d^s - p_d^m)$. Finally, note that the production decisions are independent of α so long as the household is free of the 'quota'.

The impact of α on $\widetilde{Q}_{af}$ is derived in a similar fashion as:

$$\frac{\partial \widetilde{Q}_{af}}{\partial \alpha} = (p_d^s - p_d^m)(\pi_{f,d}) - (p_d^s - p_d^m)\left((\pi_{f,v})(\pi_{v,v})^{-1}(\pi_{v,d})\right) \tag{4.2}$$

First, in a model with fixed inputs and two outputs, technical efficiency implies $\pi_{f,d}(= \pi_{d,f})$ is negative. Second, assuming V is a normal input, $\pi_{f,v}$ and $\pi_{d,v}$ are negative. Thus, with negative $(p_d^s - p_d^m)$ and positive $(\pi_{v,v})^{-1}$, both direct and virtual-price effects are positive. It means, under the circumstances, a rise in α encourages an expansion in Q_{af} production. This unambiguous result follows from technical efficiency and the presence of only two outputs. With multiple outputs it can only be said that the total production of outputs other than Q_{ad} increases.

To summarize, an increase (decrease) in α constitutes a fall (rise) in p_d and triggers a corresponding price response in output supplies. Induced by the impact on the profitability of crop enterprises resources are likely to be reallocated. This response is conditioned by the rationing in V so long as the latter binds. Comparative static analysis reveals that, as α grows, farm households prefer to switch out of crops subject to the 'quota' and into those which are not. The extent to which this preference is realized depends on technological possibilities and resource endowments. Particularly significant is the possible crop-specificity of plots and/or human and physical capital. Cultivating a crop mix less suitable to resource endowments may involve considerable yield losses or substantial adjustment costs, thereby limiting switching possibilities. Other possibilities or constraints may also come into play. For instance, a sufficiently large increase in the 'free' market price of Q_{ad} may more than compensate for a higher α such that p_d, and consequently Q_{ad},rise[47].

[46] Indeed, this demonestrates that under rationing (in V) own-price response of output declines but remains positive becuase:

$$\frac{\partial \widetilde{Q}_{ad}}{\partial p_d} = \pi_{d,d} - (\pi_{d,v})^2 (\pi_{v,v})^{-1} = \left[(\pi_{d,d})(\pi_{v,v}) - (\pi_{d,v})^2\right] (\pi_{v,v})^{-1} > 0$$

[47] Suppose both α and p_d^m have increased, then:

$$d\widetilde{Q}_{ad} = \frac{\partial Q_{ad}}{\partial p_d}\left(\frac{\partial p_d}{\partial p_d^m} dp_d^m + \frac{\partial p_d}{\partial \alpha} d\alpha\right) = \frac{\partial Q_{ad}}{\partial p_d}\left[(1-\alpha) dp_d^m - (p_d^m - p_d^s) d\alpha\right].$$

Thus, roughly speaking, a rise in p_d, and hence $\widetilde{Q}_{ad}$, occurs if:

$$(1-\alpha) dp_d^m > (p_d^m - p_d^s)\, d\alpha.$$

i.e., the rise in p_d caused by the increase in p_d^m (the term on the left) should exceed the fall in p_d triggered by the rise in α (the term on the right).

4.1.2 consumption responses

We begin by noting that, since α operates through the average price of Q_{ad} and since p_d^m (and not p_d) constitutes the opportunity cost of X_{ad}, variations in the 'quota' level can only generate an income effect on consumption. Accordingly applying (AI.17) for $\tau = \alpha$, this effect is captured by:

$$\frac{\partial \widetilde{X}_i}{\partial \alpha} = \left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*} \left(\frac{\partial X_{Mn}^h}{\partial p_n^*} \right)^{-1} \frac{\partial X_{Mn}}{\partial E} \right] \frac{\partial e'}{\partial p_d} \frac{\partial p_d}{\partial \alpha}$$

where $i = ad, af, l, Mc$. The impact of α on $\widetilde{X}_i$ thus reduces to the profit effect working directly and through the virtual price of X_{Mn}. A further simplification is achieved from noting that p_d can affect full income only through profits such that $\frac{\partial e'}{\partial p_d} = \frac{\partial \pi}{\partial p_d}$. The latter is equal to Q_{ad} by Hotelling's lemma. Using $\frac{\partial p_d}{\partial \alpha} = (p_d^s - p_d^m)$, the loss (gain) in income due to a rise (fall) in α can thus be expressed as:

$$\frac{\partial e'}{\partial \alpha} = \frac{\partial \pi}{\partial p_d} \frac{\partial p_d}{\partial \alpha} = (p_d^s - p_d^m) \widetilde{Q}_{ad}$$

Substituting, we obtain the expression for consumption responses to α :

$$\frac{\partial \widetilde{X}_i}{\partial \alpha} = (p_d^s - p_d^m) \widetilde{Q}_{ad} \left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*} \left(\frac{\partial X_{Mn}^h}{\partial p_n^*} \right)^{-1} \frac{\partial X_{Mn}}{\partial E} \right] \tag{4.3}$$

where $i = ad, af, l, Mc$.

As discussed in Appendix I the term in square brackets on the right-hand-side of (4.3) expresses the rationed income effect. Equation (4.3) shows that the reduction in income following a rise in α leads to a fall in the demand for X_i, so long as the latter is a normal good in the absence of rationing ($\frac{\partial X_i}{\partial E} > 0$). In addition, the income reduction operates as a relaxation of the ration if X_{Mn} is a normal good, subsequently lowering p_n^*. The fall in p_n^* in turn induces a further fall in the demand for X_i if X_i and X_{Mn} are net substitutes ($\frac{\partial X_i^h}{\partial p_n^*} > 0$). The opposite response will occur if the two goods are net complements ($\frac{\partial X_i^h}{\partial p_n^*} < 0$). However, an increase in α unambiguously reduces the farm households' demand for unrationed goods, so long as the latter remain normal with and without rationing ($\frac{\partial \widetilde{X}_i}{\partial E} > 0$). The reduction is an increasing function of the difference between p_d^s and p_d^m. This effect operates with or without a binding ration in X_{Mn}, so long as X_i remains normal. Under such rationing, however, the decline in demand is larger or smaller depending on whether X_i is a net substitute or a net complement to X_{Mn}, respectively.

Finally, a look at the impact of α on household welfare confirms the results obtained above. Applying (AI.11) for $\tau = \alpha$, and recalling $\frac{\partial e'}{\partial \alpha} = (p_d^s - p_d^m)\widetilde{Q}_{ad}$, leads to:

$$\frac{\partial \widetilde{U}^*}{\partial \alpha} = \left((p_d^s - p_d^m) \widetilde{Q}_{ad} \right) (e_u)^{-1} \tag{4.4}$$

The customary assumption of nonsatiation implies that $(e_u)^{-1}$ is positive. Thus a rise (fall) in α, by reducing (increasing) income, contracts (enlarges) the feasible consumption set, and as a result, unambiguously lowers (raises) household welfare. This effect gets stronger as the gap between p_d^s and p_d^m widens.

4.1.3 marketed surplus response

That the variation α affects both consumption and production means it impacts on the marketed surplus of unrationed goods which are both demanded and supplied by the farm household. Crops and labour are such commodities in the present model. The discussion below is confined to Q_{ad}, however. Analogous results apply to the others. Let S_{ad} and S_{ad}^m denote total marketed surplus of Q_{ad} and household sales of Q_{ad} on the 'free' market, respectively, i.e.:

$$\begin{aligned} \widetilde{S}_{ad} &= \widetilde{Q}_{ad} - \widetilde{X}_{ad} \; ; \\ \widetilde{S}_{ad}^m &= (1-\alpha)\widetilde{Q}_{ad} - \widetilde{X}_{ad} \end{aligned}$$

Using (4.1) and (4.3) the impact of α on $\widetilde{S}_{ad}$ can be summarized as:

$$\frac{\partial \widetilde{S}_{ad}}{\partial \alpha} = (p_d^s - p_d^m)\left\{\left[(\pi_{d,d}) - (\pi_{d,v})^2(\pi_{v,v})^{-1}\right] - \widetilde{Q}_{ad}\left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right]\right\} \tag{4.5}$$

As shown earlier a rise in α will induce a decline in both output and consumption of Q_{ad} so long as that crop is normal in the latter. Consequently, what happens to total marketed surplus depends on the relative strength of the two. In this regard, (4.5) implies that for $\widetilde{S}_{ad}$ to grow with α, the profit responsiveness of consumption should exceed the own-price responsiveness of output. Indeed, some manipulation of that equation, using the rationed counterparts of its terms directly, results the following restatement of the condition in terms of elasticities[48]:

$$\widetilde{w}_{ad}\widetilde{\xi}_{X,E} - \widetilde{\xi}_{Q,p} > 0 \tag{4.6}$$

where: $\widetilde{w}_{ad}$ = budget share of X_{ad} under rationing; $\widetilde{\xi}_{X,E}$ = rationed income elasticity of demand for X_{ad} ; $\widetilde{\xi}_{Q,p}$ = rationed own-price elasticity of Q_{ad}. If the ration constraints are not binding the unrationed analogue of (4.6) applies. Thus, total marketed surplus of Q_{ad} can increase with α only if the income elasticity of demand for X_{ad}, weighted by the corresponding budget share, is greater than the own-price elasticity of Q_{ad}.

Two observations can be made on the basis of (4.5).

1. Since the negative output and consumption effects of α apply even in the absence of rationing in V and X_{Mn}, the result concerning total marketed surplus also holds even when a household is not facing such constraints. This contrasts with the unambiguous result Azam (1992) obtained on the basis of a model with no rationing, i.e., total marketed surplus is an increasing function of the level of the 'quota'. This conclusion follows because he views the 'quota' as a lump-sum tax with the result that the possibility of an output effect does not arise in his model.

[48] Multiply (4.5), stated directly in terms of rationed effects, through by $\left(\frac{\alpha}{\widetilde{S}_{ad}}\frac{p_d}{p_d}\frac{\widetilde{Q}_{ad}}{Q_{ad}}\frac{\widetilde{X}_{ad}}{X_{ad}}\frac{E}{E}\right)$ and rearrange to obtain:

$$\widetilde{\xi}_{S,\alpha} = \left\{\left(\frac{p_d^s - p_d^m}{p_d}\right)\left(\frac{\alpha\widetilde{Q}_{ad}}{\widetilde{S}_{ad}}\right)\right\}\left[\widetilde{\xi}_{Q,p} - \widetilde{w}_{ad}\widetilde{\xi}_{X,E}\right]$$

where: $\widetilde{\xi}_{S,\alpha}$ represents the rationed elasticity of total marketed surplus with respect to α. The rest of the elasticities are as defined in the text. Since the term in curly brackets is negative, (4.6) follows.

2. The postulated pattern of rationing complicates the response of total marketed surplus in two ways. First, rationing in V weakens the own-price responsiveness of Q_{ad}. In so doing it dampens the negative impact of α on $\widetilde{S}_{ad}$. Second, through its impact on X_{ad} considered earlier, rationing in X_{Mn} increases or reduces the likelihood of $\widetilde{S}_{ad}$ rising with α if X_{ad} is a net substitute or net complement to X_{Mn}, respectively.

As to the 'free' market sales of farm households, Dercon (1994), rightly emphasizing its importance, concludes that for $\widetilde{S}^m_{ad}$ to increase with the 'quota', the following condition (stated in our notation and representing the income elasticity of demand for X_{ad} in the absence of rationing) should hold:

$$w_{ad}\xi_{X,E} > \left(\frac{p^m_d}{p^m_d - p^s_d}\right) \tag{4.7}$$

However, he uses the unrationed model of Azam (1992) and thus does not allow for the potential output effects of the 'quota' as well as the virtual-price effects due to rationing. In this regard, using (4.1) and (4.3) together with the definition of $\widetilde{S}^m_{ad}$, the following expression applies:

$$\begin{aligned}\frac{\partial \widetilde{S}^m_{ad}}{\partial \alpha} &= (p^s_d - p^m_d)\left[(1-\alpha)[(\pi_{d,d}) - (\pi_{d,v})^2(\pi_{v,v})^{-1}]\right] \\ &\quad -(p^s_d - p^m_d)\widetilde{Q}_{ad}\left[\frac{\partial X_i}{\partial E} - \frac{\partial X^h_i}{\partial p^*_n}\left(\frac{\partial X^h_{Mn}}{\partial p^*_n}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right] - \widetilde{Q}_{ad} \qquad (4.8)\end{aligned}$$

Second a condition analogous to (4.7) obtains in the present context as well. With some algebraic manipulation of (4.8), again directly using the rationed equivalents of the terms, this condition appears as [49]:

$$\widetilde{w}_{ad}\widetilde{\xi}_{X,E} - \left[(1-\alpha)\widetilde{\xi}_{Q,p} - \alpha\right] > \left(\frac{p^m_d}{p^m_d - p^s_d}\right) \tag{4.9}$$

where: $\widetilde{\xi}_{X,E}$ and $\widetilde{\xi}_{Q,p}$ are as defined above. The structure of (4.8) will be the same even if the rations do not bind, except in that case unrationed terms will replace rationed ones. Clearly (4.9) is a condition distinct from (4.7) above. Unless α exceeds $(1-\alpha)\widetilde{\xi}_{Q,p}$, it is also more stringent. Apart from the effect of rationing, (4.7) and (4.9) differ because they are based on two different characterizations of the 'quota'. Dercon (1994), following Azam (1992), considers the 'quota' as a lump-sum tax on crop producers. Equation (4.9), on the other hand, reflects the view that the 'quota' operated as a proportional tax and thus affects the production decisions of farm households. Nevertheless, both indicate that it is unlikely for compulsory grain delivery to boost 'free' market sales of $\widetilde{Q}_{ad}$.

To conclude, the introduction and expansion of CGD have increased the grain market share of the Agricultural Marketing Corporation. In contrast, a contracting 'free' market sales appears to be the more likely outcome of CGD. In fact the overall marketed surplus of Q_{ad} may have shrunk as a result. The corollary is the abolition of CGD is likely to increase this surplus. If X_{ad} and X_{Mn} are net substitutes, however, a binding ration in the latter will dampen this response. Liberalization of grain markets is a necessary but not sufficient condition for increased supply to those markets.

[49] The manipulations are the same as those used in deriving (4.7), with two differences: $\widetilde{S}^m_{ad}$ replaces $\widetilde{S}_{ad}$; and the relation $p_d = p^m_d - \alpha(p^m_d - p^s_d)$ is used.

4.2 Changes in ration levels

The model set out above contains three ration levels- R_{Mc}, R_{Mn}, and R_v. Convertible ration R_{Mc} produces an income effect in consumption through the income/subsidy associated with it. Similarly, the effect of the 'nonconvertible' ration R_{Mn}, although more complicated, is also confined to consumption. The ration level of V, on the other hand, affects production directly and consumption through farm profits. All these effects can be analyzed by suitably restricting equations (AI.13) and (AI.17) of Appendix I. However, except for a brief note on R_{Mc} at the end, the closer look presented in this section focuses on the case of R_{Mn}.

In the context of our recursive model, relaxing or tightening the rationing in X_{Mn} can only generate consumption responses. Once more (AI.17) is applied, now with $\tau = R_{Mn}$. Noting that X_i^h, X_{Mn}^h, and e' are not directly affected by R_{Mn}, the impact of a change in the level of the latter on the demand for unrationed goods is obtained as:

$$\frac{\partial \widetilde{X}_i}{\partial R_{Mn}} = \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1} + (p_n^* - p_n^s)\left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right] \quad (4.10)$$

where $i = ad, af, l, Mc$. Equation (4.10) can be interpreted as a Slutsky-type equation[50]. To illustrate, consider a relaxation of the ration constraint. The first term on the right-hand-side of (4.10) constitutes the substitution effect, and is triggered by the fall in the virtual price of X_{Mn} due to the increased level of R_{Mn}. The second term, on the other hand, captures the income effect, and is induced by the reduction in the level of expenditure required to achieve the initial level of utility that the increase in R_{Mn} brings about. The magnitude of this reduction is equal to $(p_n^* - p_n^s)$[51].

Assuming it is normal (and stays so under rationing), how household demand for X_i responds depends on the pattern of substitution in consumption. Suppose X_i and X_{Mn} are net complements, $\left(\frac{\partial X_i^h}{\partial p_n^*} < 0\right)$. Since $\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}$ - the inverse of the compensated own-price response of X_{Mn} - is negative, the substitution effect is positive. The postulated presence of excess demand for X_{Mn}, on the other hand, makes $(p_n^* - p_n^s)$ positive, which, combined with the normality of X_i, induces a positive income effect. This implies the demand for X_i rises with increased availability of the rationed good. In contrast, that the two commodities are net substitutes, $\left(\frac{\partial X_i^h}{\partial p_n^*} > 0\right)$, means the two effects work in opposite directions, with the substitution effect being negative. The resultant impact on the demand for X_i cannot be determined a priori[52]. In this

[50] This characterisation is due to Neary and Roberts (1980). In fact (4.10) is identical to equation (21) in that paper once their equations (19) and (24) are substituted in.

[51] That this is the case can be established by differentiating

$$\widetilde{e}' = e' - (p_m^* - p_m^s)R_{Mn} - (p_v^* - p_v^s)R_{Mn}$$

with respect to R_{Mn}.

[52] Notwithstanding the possibility of ambiguous specific consumption responses, the overall welfare impact of a rise in R_{Mn} is clear-cut. Using (AI.11) with $\tau = R_{Mn}$, the latter effect appears as:

$$\frac{\partial \widetilde{U}^*}{\partial R_{Mn}} = (e_u)^{-1}(p_n^* - p_n^s).$$

With the relaxation of the ration and the subsequent fall in expenditure, the household affords a less constrained choice set and a correspondingly higher utility level.

case only if the substitution effect dominates that a higher R_{Mn} engenders a lower demand for unrationed goods. For instance, household consumption of X_{ad} declines with the relaxation of the ration constraint only if it is normal and a net substitute to X_{Mn}, and if the resulting negative substitution effect exceeds the positive income effect. However, in general, with income unaffected by the change in R_{Mn}, the total demand for unrationed goods has to fall to accommodate the increased expenditure on X_{Mn}. Which of these goods is associated with the fall is what cannot be inferred a priori.

Finally, note that a change in the level of the 'convertible' ration, working through the income/subsidy related to that ration, generates income effects in consumption. Instead of examining the impact on commodity demands, here we state the welfare effect thereby engendered. Using the now familiar (AI.11) for $\tau = R_{Mc}$, the latter appears as:

$$\frac{\partial \widetilde{U}^*}{\partial R_{Mc}} = (p_c^m - p_c^s)(e_u)^{-1} \tag{4.11}$$

Since $p_c^m > p_c^s$, (4.11) shows that a rise (fall) in R_{Mc}, by increasing (decreasing) the income/subsidy associated with it, leads to a higher (lower) level of household utility[53]. Thus, ceteris paribus, abolishing the subsidized distribution of consumer goods, reduces the welfare of farm households. Obviously, this loss has to be weighted against the gains for these households that follow from the overall liberalization process, of which the abolition of subsidized consumer goods provision is just one component.

4.3 Changes in prices

There are eight exogenous prices- $p_d^m, p_f^m, p_l^m, p_c^m, p_c^s, p_n^s, p_d^s, p_v^s$ - in the model presented above. The first four are market prices, while the rest are government-determined or 'official' ones. Some of these prices affect both consumption and production decisions, whereas others impact only on the former. Table 3.1 below lists the type of effects corresponding to each. It shows that all prices generate consumption income effects of both types, whereas consumption substitution effects are confined to market prices. On the production side, recursiveness excludes the market price of 'convertible' rations from the price set relevant to production decisions, while the crop procurement price, p_d^s, is the only 'official' price affecting those decisions.

Table 3.1 also reveals that only p_d^m, p_f^m, and p_l^m are capable of initiating direct and virtual-price effects in production, as well as direct and virtual-price income and substitution effects in consumption. In that sense, and within the limits of our model, they make-up the price set most important to decisions by farm households. In recognition of this importance, a rise in one of them - p_d^m - is selected for detailed comparative static examination. The selection of p_d^m is also motivated by the desire to highlight the impact of CGD and rationing on the price responsiveness of output supplies, input demands, and commodity demands. The discussion is further restricted to the response of Q_{ad}, X_{ad}, and S_{ad}. It is straightforward to derive the effect of this and other prices on any other endogenous variable by applying (AI.13) and (AI.17) as required.

[53] In his comparison of four alternative distribution systems Sah (1987) found a similar result.

Table 2: **Effects of Exogenous prices**

Price	*Output Effect*		*Consumption Substitution Effect*		*Consumption Income Effect*	
	Direct	*Virtual-price*	*Direct*	*Virtual-price*	*Diriect*	*Virtual-price*
p_d^m	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
p_f^m	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
p_l^m	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
p_c^m	*No*	*No*	*Yes*	*Yes*	*Yes*	*Yes*
p_c^s	*No*	*No*	*No*	*No*	*Yes*	*Yes*
p_n^s	*No*	*No*	*No*	*No*	*Yes*	*Yes*
p_d^s	*Yes*	*Yes*	*No*	*No*	*Yes*	*Yes*
p_v^s	*No*	*No*	*No*	*No*	*Yes*	*Yes*

4.3.1 output responses

Like that of other exogenous variables, the impact of a change in p_d^m on outputs is worked out by using (AI.13). Here (AI.13) is applied for $\tau = p_d^m$ to give:

$$\frac{\partial \widetilde{Q}_{ad}}{\partial p_d^m} = \frac{\partial \pi_d}{\partial p_d^m} - \pi_{d,v} \left(\pi_{v,v}\right)^{-1} \frac{\partial \pi_v}{\partial p_d^m}$$

Like α, the procurement and 'free' market prices of the crop subject to CGD operate through the average price of that crop, p_d . This implies : $\frac{\partial \pi_d}{\partial p_d^m} = \frac{\partial \pi_d}{\partial p_d} \frac{\partial p_d}{\partial p_d^m}$; and $\frac{\partial \pi_v}{\partial p_d^m} = \frac{\partial \pi_v}{\partial p_d} \frac{\partial p_d}{\partial p_d^m}$. Furthermore, from the definition of p_d it follows that, $\frac{\partial p_d}{\partial p_d^m} = (1-\alpha)$. Substituting as appropriate, using the equality of cross-price effects, and multiplying the result by $(\pi_{v,v})(\pi_{v,v})^{-1}$, the desired expression is obtained as:

$$\frac{\partial \widetilde{Q}_{ad}}{\partial p_d^m} = (1-\alpha)\left[(\pi_{d,d})(\pi_{v,v}) - (\pi_{d,v})^2\right](\pi_{v,v})^{-1} \tag{4.12}$$

Three remarks can be made about supply responses on the basis of (4.12).

First, the convexity of the profit function, combined with $0 \leq \alpha \leq 1$, ensures that the overall output response is positive. The term in square brackets on the right-hand-side is positive since it is a (2×2) principal minor of the Hessian (which is positive semidefinite) of π; whereas $(\pi_{v,v})^{-1}$, being the inverse of a diagonal element of the Hessian of π, is also positive. In short, even with rationing in V, profit-maximizing behaviour on the part of farm households induces them to positively respond to rising output price.

Second, the compulsory grain delivery system reduces the own-(market)price responsiveness of Q_{ad} to a fraction, (1-α), of the level that would have obtained (for a market price equal to p_d) in its absence. The higher α is, the lower this response gets, the limit being zero at $\alpha = 1$, i.e., if the household is forced to sell the whole of its marketed surplus to the government, then Q_{ad} is unaffected by p_d^m. Imposition of the 'quota' changes the output price relevant to production choice from the market price, p_d^m, to the average price, p_d. The impact of the former flows through the latter, such that at the margin:

$$dp_d = (1-\alpha) dp_d^m.$$

Roughly speaking, out of a unit rise in p_d^m only $(1-\alpha)$ accrues to the household, which responds accordingly. By depressing the returns from Q_{ad}, the 'quota' system makes the supply of that crop, and thus total crop output, less responsive to changes in p_d^m. Furthermore, this effect operates even when rationing in V does not bind. In the latter case the output response consists only of $(1-\alpha)(\pi_{d,d})$.

Third, a binding ration in V further lowers the responsiveness of Q_{ad} to changes in its own price[54]. This effect can be isolated by carrying out the multiplication by $(\pi_{v,v})^{-1}$ in (4.12) and rearranging the resulting expression. It forms the second term of the resulting expression and appears as:

$$-\left[(1-\alpha)(\pi_{d,v})^2(\pi_{v,v})^{-1}\right]. \tag{4.13}$$

For reasons noted earlier all terms in the square bracket are positive. As a result the virtual-price effect is negative. A rise in p_d^m, pushes output of Q_{ad} higher. By increasing the demand for V (assumed to be a normal input), this operates as a tightening of the ration. The virtual price of V increases as a consequence, and, in turn, affects Q_{ad} negatively. However, as noted earlier, the total own-price effect on Q_{ad} is reduced but remains positive.

4.3.2 consumption responses

The response of the demand for X_{ad} to a change in p_d^m is derived from (AI.17). Noting that π is the only component of $Y_F^{'}$ affected by p_d^m; that p_d^m operates through p_d; and that, by Hotelling's lemma, the first-order partial derivative of π with respect to p_d is Q_{ad}, we get : $\frac{\partial Y_F^{'}}{\partial p_d^m} = (1-\alpha)Q_{ad}$. At the same time, applying Shepherd's lemma results in $\frac{\partial e}{\partial p_d^m} = X_{ad}^h$ $(= X_{ad})$. Further, symmetry of the Slutsky matrix implies that:$\frac{\partial X_{ad}^h}{\partial p_n^*} = \frac{\partial X_{Mn}^h}{\partial p_d^m}$. Substituting these results in the expression obtained from (AI.17) for $i = ad$ and $\tau = p_d^m$ leads to the desired relation:

$$\begin{aligned}\frac{\partial \widetilde{X}_{ad}}{\partial p_d^m} &= \frac{\partial X_{ad}^h}{\partial p_d^m} - \left(\frac{\partial X_{ad}^h}{\partial p_n^*}\right)^2 \left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1} \\ &\quad + [(1-\alpha)Q_{ad} - X_{ad}] \left[\frac{\partial X_{ad}}{\partial E} - \frac{\partial X_{ad}^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1} \frac{\partial X_{Mn}}{\partial E}\right]\end{aligned} \tag{4.14}$$

The direct and virtual-price substitution effects of p_d^m are respectively represented by the first two terms on the right-hand-side of (4.14). The third term combines the direct and virtual-price income effects triggered by the change in profits, $[(1-\alpha)Q_{ad}]$, and consumption expenditure, (X_{ad}). Examination of these terms generates the following observations concerning the impact of a change in p_d^m on household demand for X_{ad}[55].

1. The own-price substitution effect on X_{ad} declines in absolute terms due to rationing in X_{Mn}. The direct substitution effect is negative. If X_{ad} and X_{Mn} are net substitutes (complements), then a rise in p_d^m increases (decreases) the demand for X_{Mn}. Working as a tightening (relaxation) of the rationing constraint, this pushes the virtual price of X_{Mn} up (down). In both instances the

[54] This remark applies to the own-price responsivness of Q_{af} and L as well.
[55] Analogous remarks apply to the own-price substitution and income effects on X_l and X_{Mc}.

demand for X_{ad} rises. Compactly capturing this effect is the second term on the right-hand-side of (4.14); it is positive since $\frac{\partial X^h_{Mn}}{\partial p^*_n}$ is negative. Thus the virtual-price substitution effect is positive and consequently dampens the negative direct substitution effect. However, the overall substitution effect remains negative. Recall that all its components are second-order partial derivatives of the expenditure function, e, with respect to prices. Rewriting accordingly, multiplying by $(\frac{\partial^2 e}{\partial p^{*2}_n})(\frac{\partial^2 e}{\partial p^{*2}_n})^{-1}$, and using the notation for the rationed substitution effect we obtain:

$$\frac{\partial \widetilde{X}^h_{ad}}{\partial p^m_d} = \left[\left(\frac{\partial^2 e}{\partial p^{m2}_d}\right)\left(\frac{\partial^2 e}{\partial p^{*2}_n}\right) - \left(\frac{\partial^2 e}{\partial p^*_n \partial p^m_d}\right)^2\right]\left(\frac{\partial^2 e}{\partial p^{*2}_n}\right)^{-1} \tag{4.15}$$

The two terms on the right are 2×2 (the one in square brackets) and 1×1 principal minors of the Hessian of e. They are positive (even-order principal minor) and negative (odd-order principal minor), respectively, since e is a concave function, and thus, possesses a negative semidefinite Hessian. Therefore, under rationing in X_{Mn}, the total own-price substitution response of X_{ad} is still negative but lower in absolute terms.

2. Through the virtual-price income effect, rationing raises (lowers) the overall income effect, again depending on whether X_{ad} is a net substitute (complement) to X_{Mn} (which is assumed to be normal).

3. Compulsory grain delivery reduces the profit effect from $[Q_{ad}(\frac{\partial \widetilde{X}_{ad}}{\partial E})]$ to $[(1-\alpha)Q_{ad}(\frac{\partial \widetilde{X}_{ad}}{\partial E})]$ (where the notation for the rationed income effect is directly used for compactness). It, therefore, lowers the total income effect of p^m_d. The explanation lies in the 'quota'-constrained response of output to changes in its market price discussed earlier.

4. Assuming X_{ad} is normal in consumption, the total income effect may boost or dampen the negative impact of the substitution effect depending on whether the household is a net-buyer ($[(1-\alpha)Q_{ad} - X_{ad}] < 0$) or a net-seller ($[(1-\alpha)Q_{ad} - X_{ad}] > 0$) of Q_{ad} on the 'free' market. Thus even with the normality assumption the income effect cannot be signed a priori. This result, attributable to the profit effect, reflects the joint producer-consumer nature of agricultural households, and as such holds with or without CGD and/or rationing. The impact of rationing and CGD on the income effect has already been considered. One possible outcome of CGD can be further noted. For sufficiently high levels of α the 'quota' obligation of a household may force it to become a net-buyer on the 'free' market, thereby inducing a change in the direction of the income effect of p^m_d on its consumption demands. If a household becomes such a net buyer due to CGD, and if X_{ad} is normal in consumption and continues to be under rationing, then the income effect of p^m_d is negative and leads to a further reduction in the demand for X_{ad}.

Equation (4.14), and remarks 1-2 and 4 associated with it, illustrate that in general the total response of consumption to a rise in p^m_d is ambiguous. Even assuming a normal X_{ad} will not resolve the ambiguity in the case of net-sellers (typically a majority) of Q_{ad}. The overall impact on household welfare can, however, be determined less ambiguously. For $\tau = p_d^m$, (AI.11) results:

$$\frac{\partial \widetilde{U}^*}{\partial p_d^m} = [(1-\alpha)Q_{ad} - X_{ad}](e_u)^{-1} \tag{4.16}$$

Given nonsatiation, the only information required to sign the term on the right-hand-side relates to whether the household is a net-seller or a net-buyer of Q_{ad} on the 'free' market. Evidently, the rise in p_d^m brings about an increase in income, and subsequently welfare, to net-sellers ($[(1-\alpha)Q_{ad} - X_{ad}] > 0$). In contrast, that price rise engenders a higher expenditure (per unit of consumption), and thus a lower level of welfare, for net-purchasers ($[(1-\alpha)Q_{ad} - X_{ad}] < 0$). This is a general result in that it applies even in the absence of CGD (or $\alpha = 0$) and rationing in consumer goods.

4.3.3 marketed surplus responses

The generally ambiguous consumption responses illustrated above imply that additional information is required to sign the corresponding responses of marketed surplus of goods both produced and consumed by farm households. The exception in this regard is the case of net-buyers for which unambiguous results can be obtained. In the case of net-sellers, on the other hand, we can only identify conditions under which marketed surplus (total or net of 'quota') may rise or fall. We illustrate these points by having a closer look at the case of producers' 'free' market sales of the crop subject to CGD.

Using the definition of $\widetilde{S}_{ad}^m$ in conjunction with (4.12) and (4.14), both restated directly in terms of rationed effects, we obtain the response of marketed surplus net of 'quota' as:

$$\frac{\partial \widetilde{S}_{ad}^m}{\partial p_d^m} = (1-\alpha)^2 \frac{\partial \widetilde{Q}_{ad}}{\partial p_d} - \frac{\partial \widetilde{X}_{ad}^h}{\partial p_d^m} - [(1-\alpha)Q_{ad} - X_{ad}] \frac{\partial \widetilde{X}_{ad}}{\partial E} \tag{4.17}$$

Although reduced by CGD and rationing in V the response of output of Q_{ad} to a rise in its market price is still positive. Compensated demand for X_{ad}, on the other hand, is negatively affected by such a change. However the latter effect is dampened by rationing in X_{Mn}. Thus the first two terms on the right-hand-side of (4.17) are positive. The direction of the total response as a result depends on the sign of the third term which is the income effect. Suppose X_{ad} is normal. Then, for a net-buyer of Q_{ad}, for whom the consumption income effect is negative, net-supply to the market rises with p_d^m. Since that net-supply is negative($\widetilde{S}_{ad}^m < 0$), household net-demand for X_{ad}, which is equal to $(-\widetilde{S}_{ad}^m)$, falls. In contrast, a net-seller's supply of Q_{ad} to the 'free' market may rise or fall depending on the strength of the profit effect relative to the output effect, and the ordinary income and substitution effects in consumption. Specifically, for a rise in this net-supply the sum of the latter three effects has to be greater than the profit effect, i.e.:

$$\left((1-\alpha)^2 \frac{\partial \widetilde{Q}_{ad}}{\partial p_d} - \frac{\partial \widetilde{X}_{ad}^h}{\partial p_d^m} + X_{ad} \frac{\partial \widetilde{X}_{ad}}{\partial E} \right) > [(1-\alpha)Q_{ad}] \frac{\partial \widetilde{X}_{ad}}{\partial E}$$

Otherwise, that supply falls.

5 Conclusion

A brief characterisation of an Ethiopian farm household (and its operational milieu during the *Derg* period) indicates that, typically, such a household is simultaneously a production and consumption unit. As such it has to make optimal production and consumption choices. These choices are influenced by market variables, including prices and availability of goods and factors. They also reflect nonmarket variables such as household characteristics and production technology. The latter two sets of variables, in turn, relate to government policies, the structure and evolution of markets, the pattern of dissemination of improved technology, system of land tenure, and household resource endowments and preferences. In short, the production and consumption decisions of Ethiopian farm households and their adjustment to economic changes depend on a large number of interconnected market, technological, behavioural, and institutional factors. Clearly, it is not possible to study this complex process in its entirety. As a result it is necessary to concentrate only on some of the constituents of that process, selected on the presumption that they are important elements.

Accordingly, this paper presented a simple model of an Ethiopian farm household. The model characterizes the short-run decision-making process of such households under compulsory grain delivery and rationing in fertilizers and manufactured consumer goods. Its comparative static properties were used to explore the impact of these additional constraints on such households' choices. On the basis of the results thus derived the following observations can be made.

The compulsory grain delivery system is likely to have induced farm households to switch out of the crops that it applied to and also made the supply and own-consumption of these crops less responsive to their own prices. It also reduced the welfare of farm households by reducing their income from crops affected by it. In addition, it is likely to have depressed farm households' total marketed surplus and 'free' market sales of crops subject to the 'quota'. It is also shown that these effects are stronger, the higher the proportion of output procured and the wider the wedge between the procurement and 'free' market prices of crops falling under the CGD system. It should also be noted that CGD may have affected crop supply in ways other than inducing changes in crop-mix. For instance, the lower crop profitability engendered by CGD may adversely affect the farm households' efforts towards raising farm productivity, such as adoption of new cultivation practices and crop varieties. Or it may even have forced some of these households to reduce their dependence on crop cultivation and seek alternative income sources, such as animal husbandry. These possibilities, viable or otherwise, cannot be explicitly captured by the simple model employed in this paper. Nevertheless, on the basis of that model, it is possible to conclude that the policy of compulsory grain delivery is unlikely to have been beneficial to the growth of crop production.

In March 1990 the compulsory grain delivery system was abolished as part of an economic reform process. As far as crops previously subject to CGD are concerned, liberalization of grain markets is bound to encourage increased production; and partially restore own-price responsiveness of supply and household consumption. Furthermore it will raise farm households' incomes and, through it, boost their consumption (or demand) of goods and leisure, improving welfare as a consequence. Obviously, compared to pre-reform supply to the 'free' market, such liberalization raises crop sales to the (now unified) market since at least part of what used to be procured by the EAMC now goes through that market. But it is also likely to increase total marketed surplus above pre-reform levels.

Rationing in fertilizers, combined with excess demand for them, reduced the own-price responsiveness of output supplies and labour demand on the part of farm households. Increasing the supply of fertilizers and other modern inputs is likely to increase output and productivity of farm households. A more efficient and lasting outcome can be achieved if such increases are part of an overall agricultural sector policy which is, long-term, environmentally prudent, and appreciative of traditional farming practices.

In the context of excess demand for manufactured consumer goods, 'nonconvertible' rationing of such goods decreased the own-price substitution effect on unrationed goods including those simultaneously supplied and demanded by farm households. In so doing it may have made the total as well as net demand for such goods less responsive to changes in their own prices. On the other hand, within the limits of its applicability, 'convertible' rationing may have benefited all farm households either through subsidized consumption (relative to contemporaneous secondary market prices) or as a source of income, or both. The shift in overall government policy and the structural adjustment program adopted recently meant progressively less direct role of the state in economic activity. As part of that process provision of manufactured goods to rural households at relatively low prices was discontinued. If this decline in public sector supply is not more than matched by private sector supply, shortages of such goods may have persisted. If that is the case, it is highly probable for this persistence of shortages to dampen the impact of liberalization on marketed surplus of grain. In other words increased availability of manufactured consumer goods strengthens the positive effects of liberalization.

It is well-known that the total response of consumption to changes in prices is in general ambiguous. However, we were able to determine the overall impact on household welfare less ambiguously. The key characteristic in this regard is whether the household is a net-buyer or a net-seller of the commodity in question. Evidently, a rise in the price of a crop brings about an increase in income, and subsequently welfare, to net-sellers. In contrast, that price rise engenders a higher expenditure (per unit of consumption), and thus a lower level of welfare, for net-purchasers. The implication is that increasing crop prices do not benefit all producers.. An implication which should be incorporated in any evaluation of the welfare effects of crop price changes.

The above results show that government economic policy conditions the behaviour of farm households. Specifically, they suggest that the degree of farm households' responsiveness to change (including changes in prices and production technology) may have been hampered by CGD and rationing. The latter thus constitute two factors potentially explaining the observed unsatisfactory performance of the Ethiopian agricultural sector during the *Derg* period. As a corollary, it follows that their removal may partly explain the good performance of agriculture in recent years. In both cases, however, the explanation is likely to be partial in the light the complexity of the process which determines the performance of the sector. Important factors not explicitly considered include the weather (rainfall, in particular) and civil war. These factors directly influence outcomes in agriculture. They also compound the degree of uncertainty conditioning the behaviour of farm households. In addition to desirable changes in government policies, the recent years have been characterized by a relatively favourable weather and the absence of civil war. The latter must have contributed to the better performance of Ethiopian agriculture. Future research should extend the simple model developed in this paper in such a way that the effects of at least some of these factors are systematically examined. The study of the impact of risk, intertemporal choices and constraints, and the evolution of institutions also constitute major areas of extension.

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Appendix I

This appendix summarizes the derivation of general expressions for the comparative static responses of virtual prices, household welfare, output supplies, and commodity demands. These expressions are general in the sense that they can capture the impact of any of the exogenous variables of the model. Notations here are as in the text.

1 Response of virtual prices and utility

In this section we derive expressions for the comparative static responses of the virtual prices p_v^*, p_n^*, and utility to changes in an exogenous variable. An exogenous variable is generically denoted by τ and $\tau \in \{p_d^m, p_f^m, p_l^m, p_c^m, p_c^s, p_n^s, p_d^s, p_v^s, \alpha, R_{Mc}, R_{Mn}, R_v, T, A, K\}$. The virtual price of X_{Mn} assumes two forms depending on whether it is evaluated at a given exogenous income or utility level[56]. They are, respectively, the uncompensated virtual price (p_n^*) and the compensated virtual price $\left(p_n^{*h}\right)$. Note that the two are inverse demands. Analogous to ordinary uncompensated and compensated demands, they are equal in magnitude at equilibrium, but respond differently to exogenous variables. In other words, at a level of adjusted (virtual) exogenous income which enables the household to achieve a specified level of utility $\overline{U}$, i.e. at, $\overline{E} + (p_v^* - p_v^s)R_v + (p_n^* - p_n^s)R_{Mn} = e'$, we have (suppressing the other arguments):

$$p_n^* \left[..., e' \left(..., \overline{U} \right) \right] = p_n^{*h} \left(..., \overline{U} \right) \tag{AI.1}$$

However, working with a single representation of the virtual price simplifies presentation. One way of achieving that end follows from utility maximization. Utility maximization implies that variations in virtual exogenous income, caused by movements in an exogenous variable τ, will induce corresponding changes in the utility level achieved. In order to allow for this impact we substitute the indirect utility function U^* in the place of $\overline{U}$ in (AI.1). This substitution enables us to exclusively use p_n^* below.

To derive the effect of exogenous changes on p_v^*, p_n^* and U^*, let us begin by restating the equilibrium of the household as (AI.2)-(AI.4), which respectively represent the two rationing constraints and the income-expenditure equality. In so doing we have applied Shepherd's lemma to obtain $\frac{\partial e'}{\partial p_n^*} = X_{Mn}^h$ and $\frac{\partial e'}{\partial p_v^*} = V$.

$$e'_v = R_v \tag{AI.2}$$

$$e'_n = R_{Mn} \tag{AI.3}$$

$$e'(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, U^*) = \overline{E} + (p_v^* - p_v^s)R_v + (p_n^* - p_n^s)R_{Mn} \tag{AI.4}$$

[56] As noted above, in the present case, the ordinary exogenous income of the farm household is adjusted by the income/subsidy obtained from convertible rations. Note that this income/subsidy is exogenous since both prices, p_c^m and p_c^s, as well as the ration level, R_{Mc}, do not depend on household choices. The resulting income is referred to as adjusted exogenous income.

where : $\overline{E} = E + (p_c^m - p_c^s)R_{Mc}$. Primarily, this restatement is useful in that it explicitly accommodates the interdependence between p_n^* and U^*. It also enables us to directly examine the impact of exogenous variables on household welfare. Totally differentiating (AI.2)-(AI.4):

$$e'_{v,\tau}d\tau + e'_{v,v}dp_v^* + e'_{v,n}dp_n^* + e'_{v,u}dU^* = \frac{\partial R_v}{\partial \tau}d\tau$$

$$e'_{n,\tau}d\tau + e'_{n,v}dp_v^* + e'_{n,n}dp_n^* + e'_{n,u}dU^* = \frac{\partial R_{Mn}}{\partial \tau}d\tau$$

$$\begin{aligned} e'_{\tau}d\tau + e'_{v}dp_v^* + e'_{n}dp_n^* + e'_{u}dU^* &= \frac{\partial \overline{E}}{\partial \tau}d\tau + R_v dp_v^* + (p_v^* - p_v^s)dR_v - R_v dp_v^s \\ &\quad + R_{Mn}dp_n^* + (p_n^* - p_n^s)dR_{Mn} - R_{Mn}dp_n^s \end{aligned}$$

where: $e'_\tau = \frac{\partial e'}{\partial \tau}$; $e'_v = \frac{\partial e'}{\partial p_v^*}$; $e'_n = \frac{\partial e'}{\partial p_n^*}$; $e'_u = \frac{\partial e'}{\partial U}$; $e'_{v,\tau} = \frac{\partial^2 e'}{\partial \tau \partial p_v^*}$; $e'_{v,u} = \frac{\partial^2 e'}{\partial U \partial p_v^*}$; $e'_{v,v} = \frac{\partial^2 e'}{\partial p_v^{*2}}$; $e'_{n,\tau} = \frac{\partial^2 e'}{\partial \tau \partial p_n^*}$; $e'_{n,n} = \frac{\partial^2 e'}{\partial p_n^{*2}}$; and $e'_{n,u} = \frac{\partial^2 e'}{\partial U \partial p_n^*}$ are the first- and second-order partial derivatives of the expenditure function with respect to the corresponding variables. Noting that:

$$e'_v = -\pi_v = V = R_v$$

$$e'_n = e_n = X_{Mn}^h = R_{Mn}$$

$$\frac{\partial \overline{E}}{\partial \tau} = R_{Mc}\frac{\partial p_c^m}{\partial \tau} + (p_c^m - p_c^s)\frac{\partial R_{Mc}}{\partial \tau} - R_{Mc}\frac{\partial p_c^s}{\partial \tau}; \text{ since } \frac{\partial E}{\partial \tau} = 0$$

and rearranging[57]:

$$e'_{v,v}dp_v^* + e'_{v,n}dp_n^* + e'_{v,u}dU^* = -\left[e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}\right]d\tau \qquad \text{(AI.5)}$$

$$e'_{n,v}dp_v^* + e'_{n,n}dp_n^* + e'_{n,u}dU^* = -\left[e'_{n,\tau} - \frac{\partial R_{Mn}}{\partial \tau}\right]d\tau \qquad \text{(AI.6)}$$

$$\begin{aligned} e'_u dU^* &= \{R_{Mc}\frac{\partial p_c^m}{\partial \tau} + (p_c^m - p_c^s)\frac{\partial R_{Mc}}{\partial \tau} - R_{Mc}\frac{\partial p_c^s}{\partial \tau} - V\frac{\partial p_v^*}{\partial \tau} + R_v\frac{\partial p_v^*}{\partial \tau} \\ &\quad + (p_v^* - p_v^s)\frac{\partial R_v}{\partial \tau} - R_v\frac{\partial p_v^s}{\partial \tau} + R_{Mn}\frac{\partial p_n^*}{\partial \tau} - X_{Mn}^h\frac{\partial p_n^*}{\partial \tau} + (p_n^* - p_n^s)\frac{\partial R_{Mn}}{\partial \tau} \\ &\quad - R_{Mn}\frac{\partial p_n^s}{\partial \tau} - e'_\tau\}d\tau \end{aligned} \qquad \text{(AI.7)}$$

Divide through by $d\tau$ and rearrange using $V = R_v$ and $R_{Mn} = X_{Mn}^h$. Furthermore, we exploit the following results:

1) That $e'(.)$ is twice-continuously differentiable implies that it has, by Young's theorem, a symmetric Hessian. Thus, $e'_{v,n} = e'_{n,v}$.

2) The separability of production and consumption decisions imply that p_n^* and U^* do not affect the demand for V, and that p_v^* does not influence the compensated demand X_{Mn}^h.

[57] Note that: $\frac{\partial R_{Mc}}{\partial \tau} = 0$ if $\tau \neq R_{Mc}$; $\frac{\partial p_c^m}{\partial \tau} = 0$ if $\tau \neq p_c^m$; $\frac{\partial p_c^s}{\partial \tau} = 0$ if $\tau \neq p_c^s$; $\frac{\partial R_v}{\partial \tau} = 0$ if $\tau \neq R_v$; $\frac{\partial p_v^s}{\partial \tau} = 0$ if $\tau \neq p_v^s$; $\frac{\partial R_{Mn}}{\partial \tau} = 0$ if $\tau \neq R_{Mn}$; and $\frac{\partial p_n^s}{\partial \tau} = 0$ if $\tau \neq p_n^s$.

Therefore, $e'_{v,n} = e'_{n,v} = e'_{v,u} = 0$. The resulting three equations can be restated in matrix form as:

$$\begin{bmatrix} e'_{v,v} & 0 & 0 \\ 0 & e'_{n,n} & e'_{n,u} \\ 0 & 0 & e'_u \end{bmatrix} \begin{bmatrix} \frac{\partial p_v^*}{\partial \tau} \\ \frac{\partial p_n^*}{\partial \tau} \\ \frac{\partial U^*}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -(e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}) \\ -(e'_{n,\tau} - \frac{\partial R_{Mn}}{\partial \tau}) \\ \sum_k [(p_k^* - p_k^s)\frac{\partial R_k}{\partial \tau} - R_k \frac{\partial p_k^s}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau \end{bmatrix} \quad \text{(AI.8)}$$

where, $k = Mn, v$; and $\frac{\partial \overline{E}}{\partial \tau}$ is as defined above.

Using Cramer's Rule we obtain the expression for the response of the virtual price p_v^* to changes in an exogenous variable τ[58]:

$$\frac{\partial p_v^*}{\partial \tau} = \left(e'_{v,v} e'_{n,n} e'_u\right)^{-1} \det \begin{bmatrix} -(e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}) & 0 & 0 \\ -(e'_{n,\tau} - \frac{\partial R_{Mn}}{\partial \tau}) & e'_{n,n} & e'_{n,u} \\ \sum_k [(p_k^* - p_k^s)\frac{\partial R_k}{\partial \tau} - R_k \frac{\partial p_k^s}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau & 0 & e'_u \end{bmatrix}$$

where det = determinant. Expanding and simplifying:

$$\begin{aligned} \frac{\partial p_v^*}{\partial \tau} &= \left(e'_{v,v} e'_{n,n} e'_u\right)^{-1} \left(e'_{n,n} e'_u\right) \left(e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}\right) \\ &= (e'_{v,v})^{-1} \left(e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}\right) \end{aligned}$$

Since $e'_{v,v} = -\frac{\partial^2 \pi}{\partial p_v^{*2}} = -\pi_{v,v}$; and $e'_{v,\tau} = -\frac{\partial^2 \pi}{\partial \tau \partial p_v^*} = -\pi_{v,\tau}$ [where $\pi_{v,i}$ are second-order partial derivatives of the unrationed profit function (at virtual prices) with respect to variable i], we finally obtain:

$$\frac{\partial p_v^*}{\partial \tau} = -\left(\pi_{v,v}\right)^{-1} \left(\pi_{v,\tau} + \frac{\partial R_v}{\partial \tau}\right) \quad \text{(AI.9)}$$

Due to separability, p_v^* is exclusively determined by production-side variables. Consequently $p_c^m, p_c^s, p_n^s, R_{Mc}$, and R_{Mn} do not affect p_v^*. Furthermore, since production and consumption decisions are separable, and since it affects the amount of compensation required due the imposition of the ration R_v on the household alone, p_v^s does not influence either p_v^* or optimal production choices[59].

Following analogous steps the impact of τ on p_n^* is obtained. Cramer's rule implies that:

$$\frac{\partial p_n^*}{\partial \tau} = \left(e'_{v,v} e'_{n,n} e'_u\right)^{-1} \det \begin{bmatrix} e'_{v,v} & -(e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}) & 0 \\ 0 & -(e'_{n,\tau} - \frac{\partial R_{Mn}}{\partial \tau}) & e'_{n,u} \\ 0 & \sum_k [(p_k^* - p_k^s)\frac{\partial R_k}{\partial \tau} - R_k \frac{\partial p_k^s}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau & e'_u \end{bmatrix}$$

After some algebraic manipulation, this expression simplifies to:

$$\frac{\partial p_n^*}{\partial \tau} = -(e'_{n,n})^{-1} \left(e'_{n,\tau} - \frac{\partial R_{Mn}}{\partial \tau}\right) - (e'_{n,n} e'_u)^{-1} \left[\sum_k [(p_k^* - p_k^s)\frac{\partial R_k}{\partial \tau} - R_k \frac{\partial p_k^s}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau\right] \quad \text{(AI.10)}$$

[58] To obtain well-defined inverses we slightly strengthen the standard concavity property of the expenditure function to strict concavity in virtual prices. In other words we assume that the demand for each of the rationed goods always responds to changes in the corresponding own-virtual price.

[59] In other words, p_v^* and optimal production choices are independent of household income.

The first term on the right-hand-side of (AI.10) represent the response of the virtual price at constant utility, i.e., the substitution effect, while the second constitutes the income effect.

Similarly applying Cramer's Rule to AI.8, the change in utility is derived as:

$$\frac{\partial U^*}{\partial \tau} = \left(e'_{v,v} e'_{n,n} e'_u\right)^{-1} \det \begin{bmatrix} e'_{v,v} & 0 & -(e'_{v,\tau} - \frac{\partial R_v}{\partial \tau}) \\ 0 & e'_{n,n} & -(e'_{n,\tau} - \frac{\partial R_{Mn}}{\partial \tau}) \\ 0 & 0 & \sum_k [(p^*_k - p^s_k)\frac{\partial R_k}{\partial \tau} - R_k \frac{\partial p^s_k}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau \end{bmatrix}$$

Expanding and simplifying, we get:

$$\frac{\partial U^*}{\partial \tau} = (e'_u)^{-1} \left[\sum_k [(p^*_k - p^s_k)\frac{\partial R_k}{\partial \tau} - R_k \frac{\partial p^s_k}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau \right]. \tag{AI.11}$$

Equation (AI.11) neatly summarizes the impact of an exogenous variable on unrationed utility at virtual prices, U^*, and thus the optimal level of utility under rationing, $\widetilde{U}^*$, since the latter is equal to the former. This property is desirable if the primary interest is to determine the overall effect of τ on household welfare. In some instances it may also generate simpler and/or unambiguous results even when specific consumption responses are complicated and/or indeterminate a priori.

2 Production and consumption responses

2.1 production responses

Recall that rationed supplies, $\widetilde{Q}_{aj}, (j = d, f)$, and unrationed supplies, $Q_{aj}, (j = d, f)$ at virtual prices are equal. Recursiveness implies that only production-side variables affect production choices. Further, by Hotelling's lemma, each is equal to the first-order derivative of the corresponding profit function with respect to the relevant output price. Thus:

$$\widetilde{Q}_{aj} = \frac{\partial}{\partial p_j} \widetilde{\pi} \left(p_d, p^m_f, p_l, p^s_v, \rho, R_v, A, K\right) = \widetilde{\pi}_j$$

$$Q_{aj} = \frac{\partial}{\partial p_j} \pi \left(p_d, p^m_f, p_l, p^*_v, \rho, A, K\right) = \pi_j$$

and

$$\widetilde{Q}_{aj} = Q_{aj} = \pi_j. \tag{AI.12}$$

where: $p_j \in \{p_d, p^m_f\}$; $j = d, f$; and as in the text, $\widetilde{\pi}$ = rationed profit function, and π = unrationed profit function at virtual prices. Accordingly the response of rationed outputs to changes in exogenous variables can be analyzed in terms of the corresponding unrationed outputs at virtual prices. Towards that end differentiate AI.12 with respect to τ, allowing for changes in p^*_v in doing so :

$$\frac{\partial \widetilde{Q}_{aj}}{\partial \tau} = \pi_{j,\tau} + \pi_{j,v} \frac{\partial p^*_v}{\partial \tau}$$

where: $\pi_{j,\tau} = \frac{\partial^2 \pi}{\partial \tau \partial p_j}; \pi_{j,v} = \frac{\partial^2 \pi}{\partial p^*_v \partial p_j}$. Substituting for the response of p^*_v from AI.9:

$$\frac{\partial \widetilde{Q}_{aj}}{\partial \tau} = \pi_{j,\tau} - \pi_{j,v}\left(\pi_{v,v}\right)^{-1}\left(\pi_{v,\tau} + \frac{\partial R_v}{\partial \tau}\right);\ j = d, f \qquad \text{(AI.13)}$$

The first term on the right-hand-side of AI.13 captures the direct effect of a change in τ, while the second constitutes the impact through the virtual price of V. Similarly, noting the equality of rationed and unrationed labour demand and applying Hotelling's lemma:

$$\widetilde{L} = L = -\pi_l$$

Differentiating with respect to τ and substituting for $\frac{\partial p_v^*}{\partial \tau}$, the response of demand for labour is obtained:

$$\frac{\partial \widetilde{L}}{\partial \tau} = -\left[\pi_{l,\tau} - \pi_{l,v}\left(\pi_{v,v}\right)^{-1}\left(\pi_{v,\tau} + \frac{\partial R_v}{\partial \tau}\right)\right] \qquad \text{(AI.14)}$$

where: $\pi_{l,\tau} = \frac{\partial^2 \pi}{\partial \tau \partial p_l^m}; \pi_{l,v} = \frac{\partial^2 \pi}{\partial p_v^* \partial p_l^m}$.

2.2 consumption responses

In deriving expressions for consumption responses we make use of the following results obtained earlier.

- At, $\overline{E} = \widetilde{e'}(.)$, rationed Marshallian and Hicksian demands are equal:

  $$\widetilde{X}_i\left[p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, \widetilde{e'}(.)\right] = \widetilde{X}_i^h\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, R_{Mn}, \overline{U}\right)$$

  Analogously, unrationed Marshallian and Hicksian demands at virtual prices are equal to each other at, $\overline{E} + (p_v^* - p_v^s)R_v + (p_n^* - p_n^s)R_v = e'(.)$:

  $$X_i\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, e'(.)\right) = X_i^h\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, \overline{U}\right)$$

  where:

  $X_i \in (X_{ad},\ X_{af},\ X_l,\ X_{Mc},\ X_{Mn})$;
  $e'(.) = e'\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, \overline{U}\right)$; and
  $\widetilde{e'}(.) = \widetilde{e'}\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, \overline{U}\right)$.

- Rationed Marshallian demands are equal to unrationed Marshallian demands evaluated at virtual prices and virtual exogenous income adjusted by the income/subsidy from convertible rations:

$$X_i(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, p_v^s, \rho, R_{Mn}, R_v, T, A, K, R_{Mn}, \overline{E}) = X_i[p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, p_v^*, \rho, T, A, K, \overline{E}']$$

- where $\overline{E}' = \overline{E} + (p_v^* - p_v^s)R_v + (p_n^* - p_n^s)R_{Mn}$. Further, at a given utility level, $\overline{U}$, rationed Hicksian demands and unrationed Hicksian demands at virtual prices are equal:

  $$\widetilde{X}_i^h\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^s, R_{Mn}, \overline{U}\right) = X_i^h\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, \overline{U}\right).$$

- By Shepherd's lemma:

$$\frac{\partial e}{\partial p_i} = e_i = X_i^h\left(p_d^m, p_f^m, p_l^m, p_c^m, p_n^*, \overline{U}\right)$$

where $p_i \in \left\{p_d^m, p_f^m, p_l^m, p_c^m, p_n^*\right\}$.

>From these relations it follows that at appropriately defined income and expenditure levels:

$$\widetilde{X}_i = X_i = e_i. \tag{AI.15}$$

Allowing for the endogeneity of p_n^* and $\overline{U}$ we differentiate AI.15 with respect to τ we obtain:

$$\frac{\partial\ \widetilde{X}_i}{\partial\tau} = e_{i,\tau} + e_{i,n}\frac{\partial p_n^*}{\partial\tau} + e_{i,u}\frac{\partial U^*}{\partial\tau}.$$

Substituting for the response of p_n^* and U^* from AI.10-AI.11, and collecting terms:

$$\begin{aligned}\frac{\partial\ \widetilde{X}_i}{\partial\tau} &= e_{i,\tau} - e_{i,n}\left(e_{n,n}\right)^{-1}\left(e_{n,\tau} - \frac{\partial R_{Mn}}{\partial\tau}\right) \\ &\quad +(e_u)^{-1}\left(e_{i,u} - e_{i,n}(e_{n,n})^{-1}e_{n,u}\right)\left[\sum_k\left[(p_k^* - p_k^s)\frac{\partial R_k}{\partial\tau} - R_k\frac{\partial p_k^s}{\partial\tau}\right] + \frac{\partial\overline{E}}{\partial\tau} - e'_\tau\right]\end{aligned} \tag{AI.16}$$

The nature of this expression becomes apparent if we note the following.

- Exploiting the equality of unrationed Marshallian and Hicksian demands let us differentiate both with respect to u. Rearrange the result using, $X_i^h = e_i$ and $e'_u = e_u$ [since, by the separability of production and consumption decisions and the exogeniety of p_l^m and T, $\frac{\partial}{\partial U}(\pi + p_l^m T) = 0$], we obtain the unrationed income effect on demands[60,61]:

$$\frac{\partial\ X_i}{\partial E} = e_{i,u}(e_u)^{-1}$$

Similarly:

$$\frac{\partial X_{Mn}}{\partial E} = e_{n,u}(e_u)^{-1}$$

[60] With a slight abuse of notation the income derivative of demand for goods is represented by $\frac{\partial X_i}{\partial E}$ for all forms of income.

[61] The duality between utility maximization and expenditure minimization implies that the inverse of the marginal cost of utility, $(e_u)^{-1}$, is equal to the marginal utility of income , $\frac{\partial U^*}{\partial E}$. Thus we can also obtain the result by differentiating $X_i = e_i(..., U^*)$ with respect to E:

$$\frac{\partial X_i}{\partial E} = e_{i,u}\frac{\partial U^*}{\partial E} = e_{i,u}(e_u)^{-1}.$$

where $e_u = \frac{\partial e}{\partial U}$. For further details regarding the relation between $\frac{\partial U^*}{\partial E}$ and $(e_u)^{-1}$ see Deaton and Muellbauer (1980); and Gravelle and Rees (1992).

- Shepherd's lemma implies that:

$$e_{i,\tau} = \frac{\partial\ X_i^h}{\partial \tau}$$

$$e_{n,\tau} = \frac{\partial\ X_{Mn}^h}{\partial \tau}$$

$$e_{i,n} = \frac{\partial\ X_i^h}{\partial p_n^*}$$

$$e_{n,n} = \frac{\partial\ X_{Mn}^h}{\partial p_n^*}$$

The last two mainly notational results enable us to rewrite AI.15 in the conventional comparative statics format:

$$\begin{aligned}\frac{\partial \widetilde{X}_i}{\partial \tau} &= \left\{\frac{\partial X_i^h}{\partial \tau} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\left(\frac{\partial X_{Mn}^h}{\partial \tau} - \frac{\partial R_{Mn}}{\partial \tau}\right)\right\} \\ &+ \left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right]\left[\sum_k [(p_k^* - p_k^s)\frac{\partial R_k}{\partial \tau} - R_k\frac{\partial p_k^s}{\partial \tau}] + \frac{\partial \overline{E}}{\partial \tau} - e'_\tau\right]\end{aligned} \tag{AI.17}$$

The term in curly brackets on the right-hand-side of AI.17 represent the substitution effect, while the second (the product of the terms in the two square brackets) expresses the income effect. Unlike their standard counterparts, each of these has direct and indirect or virtual-price components. Also greater is the number of sources triggering these effects. These special features warrant a closer look. First consider the substitution effect. It is composed of: (i) the direct (ordinary) substitution effect, $\frac{\partial X_i^h}{\partial \tau}$, which is the unrationed substitution effect; and (ii) the virtual-price substitution effect capturing the impact of rationing on substitution possibilities in consumption:

$$\frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\left(\frac{\partial X_{Mn}^h}{\partial \tau} - \frac{\partial R_{Mn}}{\partial \tau}\right).$$

Note also that, at constant utility, variations in p_n^* may follow either directly from changes in the ration level ($\tau = R_{Mn}$), or indirectly from changes in the unrationed Hicksian demand for the rationed good, induced by τ ($\tau \neq R_{Mn}$). Finally, that rationed Hicksian and unrationed Hicksian demands are equal once the latter are evaluated at virtual prices means that:

$$\frac{\partial \widetilde{X}_i^h}{\partial \tau} = \frac{\partial X_i^h}{\partial \tau} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\left(\frac{\partial X_{Mn}^h}{\partial \tau} - \frac{\partial R_{Mn}}{\partial \tau}\right) \tag{AI.18}$$

AI.17 is analogous to Equation 29 of Neary and Roberts(1980:34), but defined for a general exogenous variable τ. It shows that the rationed substitution effect, $\frac{\partial \widetilde{X}_i^h}{\partial \tau}$, differs in absolute magnitude from its unrationed counterpart. The direction and size of the difference depend on the specific exogenous variable considered and, on whether X_i and X_{Mn} are substitutes or complements.

Similar remarks apply to the income effect. First consider a comparison with the pure consumer case. In this rationed environment, a change in exogenous income, $\overline{E}$, produces a direct (ordinary) effect, $\frac{\partial X_i}{\partial E}$, representing the unrationed income effect. It also leads to a virtual-price income effect as captured by, $\frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}$. This decomposition, combined with the equality of rationed Marshallian demands and unrationed Marshallian demands at virtual prices, implies the following relation between the rationed and unrationed income effects:

$$\frac{\partial \widetilde{X}_i}{\partial E} = \frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}. \tag{AI.19}$$

This expression is identical to Equation 24 of Neary and Roberts (1980) (once their Equation 19 is substituted) derived for a pure consumer. It implies that, under rationing, a change in $\overline{E}$, in addition to its ordinary effect, equivalently operates as a change in the ration level R_{Mn} and produce a corresponding effect on the demand for unrationed goods. The direction and extent of the latter effect depend on whether X_{Mn} is normal and whether X_i and X_{Mn} are net substitutes or complements. For instance, if X_{Mn} is normal and a net substitute to a normal X_i, a rise in $\overline{E}$ also works as a tightening of the ration thereby increasing the virtual price p_n^*. The latter in turn induce a positive substitution-like effect on the demand for X_i further strengthening the positive income effect. In this particular example, therefore, the rationed income effect exceeds its unrationed counterpart.

That the farm household is a producer-consumer, and that ration levels as well as the prices at which they are acquired may change, introduce additional avenues through which the income effect operates. Including the usual change in expenditure (or real income), we can identify three such avenues that a change in τ may induce:

- a change in net expenditure (or real net income), e'_τ, which has two components, namely, a change in total expenditure (or e_τ) and a change in full income (or $\frac{\partial}{\partial \tau}(\pi + p_l^m T)$)[62];
- a change in exogenous income, $\frac{\partial \overline{E}}{\partial \tau}$; and
- a change in the compensation required to make the household choose the ration levels voluntarily, $\left[(p_v^* - p_v^s)\frac{\partial R_v}{\partial \tau} - R_v\frac{\partial p_v^s}{\partial \tau} + (p_n^* - p_n^s)\frac{\partial R_{Mn}}{\partial \tau} - R_{Mn}\frac{\partial p_n^s}{\partial \tau}\right]$.

Thus the overall income effect comprises the *ordinary income effect*

$$\left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right] e_\tau;$$

[62] Recall that $e'(.) = e(.) - [\pi(.) + p_l^m T]$. Thus:

$$e'_\tau = \frac{\partial e'(.)}{\partial \tau} = \frac{\partial e(.)}{\partial \tau} - \left[\frac{\partial \pi(.)}{\partial \tau} + \frac{\partial p_l^m T}{\partial \tau}\right]$$

the *profit effect*[63]

$$\left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right]\frac{\partial}{\partial \tau}(\pi + p_l^m T);$$

and what can be called the *compensation effect*

$$\left[\frac{\partial X_i}{\partial E} - \frac{\partial X_i^h}{\partial p_n^*}\left(\frac{\partial X_{Mn}^h}{\partial p_n^*}\right)^{-1}\frac{\partial X_{Mn}}{\partial E}\right]\left[(p_v^* - p_v^s)\frac{\partial R_v}{\partial \tau} - R_v\frac{\partial p_v^s}{\partial \tau} + (p_n^* - p_n^s)\frac{\partial R_{Mn}}{\partial \tau} - R_{Mn}\frac{\partial p_n^s}{\partial \tau}\right].$$

Obviously which of these effects actually occur depends on the specific exogenous variable being considered.

[63] Strictly speaking, this effect has to be referred to as the *full income effect* since, in addition to profits, the value of the household's time endowment will change if $\tau \in (p_l^m, T)$. However, in line with tradition, we refer to it as the profit effect.