# Skill-biased technology imports, improved education access and rising wage inequality in a developing country

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**Abstract**

As a response to skill-biased technological change (SBTC), expanded education access is regarded by many as an appropriate way to reduce wage inequality, so developing countries are engaged in a race between education and SBTC. In this model, developing countries (the South) adopt technologies by purchasing licences from technology leaders (the North). If skilled and unskilled labour are perfect substitutes, we show tehnological change is skill-biased in the South simply because it is in the North, resulting in permanently rising wage inequality in the South. We model expanded educational access as producing relatively educated new cohorts of labour market entrants. This makes the market for skill-biased technologies more attractive, which generates accelerated wage inequality. Allowing for skilled and unskilled labour to be imperfect substitutes shows the elasticity of substitution ($\sigma$) would have to be high for a rise in skill supply to raise wage inequality. The threshold of $\sigma$ required is higher than for developed countries and higher than for Northern skill-biased technological change to do so. Existing estimates of $\sigma$ suggest greater skill supply is unlikely to increase wage inequality and it is more likely that global technology patterns will.

# 1 Introduction

It is well documented that, despite a steady rise in the supply of skilled workers in the United States, wage inequality increased in the second half of the 20th century. There is also evidence of shifts in labour demand that favour skilled workers in other OECD countries. A consensus has emerged that skill-biased technological change (SBTC) has been the major cause of the rise in wage inequality (Katz & Autor, 1999). There is evidence that developing countries, particularly middle income countries, have also experienced technical change that favours skilled workers (Berman & Machin, 2000) and that some have seen rises in wage inequality (Berman, Bound and Machin, 1998).

One[1] explanation for SBTC is that, because incentive driven researchers determine whether to devote resources to discovering skill-biased technologies or those that favour unskilled workers, the degree of skill bias is endogenous or "directed" (Acemoglu, 2002ab; Kiley, 1999). The directed technological change literature suggests the rise in the supply of skills in the UK and USA made it relatively more profitable to produce skill-biased technologies, which raise the relative productivity of skilled workers. Under certain circumstances, this effect can dominate the traditional substitution effect and result in a rise in wage inequality.

Caselli & Coleman (2000) develop a model in which countries have a choice of technology options subject to a technology frontier; the degree of skill-bias in the technologies chosen depends on relative skill endowments. From a sample of a wide variety of countries, they present a strong correlation between relative factor shares and relative factor productivities. To the extent that factor productivity is an indicator of the technologies available to that factor and factor shares indicate the attractiveness of the market, this evidence is consistent with the view that the skill bias of technology is directed in both rich and poor countries. Just like in the North, the relative attractiveness of skill-biased technologies will influence the skill-bias of technology adoption and could lead to a rise in the skill premium in developing countries.

On the other hand, we propose that the skill-bias of technologies in developing countries can be directed by developments in the North. Berman, Bound and Machin (1998) argue that SBTC is pervasive across the globe, affecting both OECD and developing countries. Furthermore, most of the developing countries in their sample experienced increases in the skill premium and in the skilled share of employment. Savvides & Zachariadis (2005) use manufacturing data to find that developing countries undertake no own R&D but rely on foreign technology transfer. Berman & Machin (2000) also argue that technology adoption in the South is driven by that in the North.[2] They find that the same industries experiencing SBTC in the South in the 1980s were those experiencing it in the North in prior decades.

We therefore have evidence that skill bias in developing countries is influenced both by external skill biased technical progress and by domestic conditions like relative skill supply. This paper builds a model that captures both these features. In keeping with existing work on directed technological change, we base our explanation on an endogenous growth model.

Initially assuming skilled and unskilled labour are perfect substitutes, as in Kiley (1999), we present three departures from the existing literature. First, instead of firms in a technology sector paying to develop technologies, we have firms in an import sector that acquire the licence for a new product from abroad in exchange for units of output exported. Given that the overwhelming proportion of new technologies are developed in only a handful of countries, this process of technology adoption is arguably more relevant to the vast majority of countries, particularly those in the developing world. Second, we assume that skill-biased technologies advance faster than technologies complementing unskilled labour in the technologically advanced countries. We call this Northern skill-biased technological change (NSBTC). NSBTC will feed directly into the pace at which technologies are imported and adopted by the South:

[1] For alternative explanations and a critique of this view, see Sanders (2005) and Krussell, Ohanian, Rios-Rull & Violante (2000).

[2] This finding is limited to middle income countries, not the least developed countries in their sample.

developing countries in the South have skill-biased technological progress because the North does. In our model, this means there can be a rise in wage inequality simply because of international technology patterns.

Third, in addition to one-off changes in skill supply, we allow for a gradual change in the skill composition. To capture the effects of education reforms on the entire population, we model changes in the proportion of cohorts being educated as they enter the labour force and replace those cohorts that die. This generates periods of growth in the skill composition of the population. We see potentially long periods of accelerated growth in wage inequality. We also show that the unskilled lose not only relative to skilled workers, but that they receive lower wages than if there had been no rise in skill supply.

The Global Competitiveness Report views an increase in the skills base, be it through expansion of both primary and higher education, as a key ingredient for competitiveness (Lopez-Claros, Altinger, Blanke, Drzeniek & Miaet, 2006). This may be a response to global labour demand shifts but may itself lead to further skill-biased labour demand shifts. To the extent that expanded educational access is seen as an instrument against wage inequality in developing countries, the perfect substitutes model suggests it would be self-defeating. Tinbergen (1975:79) warned of a *"race between education and technology"*. If education increases rather than decreases wage inequality, education is in fact running backwards in the race. The insights from the model imply education is.running backwards.

However, existing literature suggests the assumption of perfect substitutability is important for the final conclusion that a rise in skill supply leads overall to a rise in wage inequality (for example Acemoglu, 2002ab), so we proceed to relax it to understand how important such an assumption might be.

To focus on the effects of imperfect substitutability, we concentrate on steady-state equilibria, where the skill supply and skill bias of the world technology frontier are constant. We conduct a comparative static analysis to find that a one-off rise in skill supply almost unequivocally leads to SBTC. We also find that a one-off rise in the ratio of the stock of the availability of skilled machines relative to unskilled machines, which we also interpret as Northern skill-biased technological change, can lead to SBTC.

The most important qualification introduced by relaxing the perfect substitutability assumption is that, for a rise in skill supply to lead to a rise in wage inequality, a high elasticity of substitution ($\sigma$) is needed. The threshold of $\sigma$ exceeds that for developed county models (Acemoglu, 2002a). The parameter values required for NSBTC to raise domestic wage inequality are not onerous, making it a more likely cause of increased wage inequality. Finally, the parameter requirements are compared with existing parameter estimates for developing countries. Available estimates suggest wage inequality would not go up after a rise in skill supply while the results are ambiguous after NSBTC. Thus, it appears that governments' use of education as a response to global skill-biased technology patterns would be an appropriate response. The race between education and the skill-bias of technology is one worth running.

Section 2 assumes perfect substitutability and holds the skill composition constant in order to describe the model. Section 3 shows the effects of a rising skill supply assuming perfect substitutability between skilled and unskilled labour. Section 4 presents the qualifications introduced by relaxing the assumption of perfect elasticity and compares the model with existing data and parameter estimates and Section 5 concludes.

## 2 Model description

We begin by assuming skilled and unskilled labour are perfect substitutes, relaxing this assumption in section 4. In this section, we describe the model assuming a constant skill composition. Producers use skilled labour (and a variety of machines that complement skilled labour) as well as unskilled labour (and machines that complement unskilled labour) to produce final output. Any agent can decide to acquire a licence from a technologically advanced nation. The licence carries the exclusive right to import and distribute a particular technology, which works with either skilled or unskilled labour. The decision to acquire a licence will depend on the domestic skill composition, which affects the potential revenues from a licence and will depend on how advanced technologies are abroad, which affects the cost of a licence. We will show how the skilled (unskilled) wage is directly proportional to the number of skilled (unskilled) machine varieties available.

## 2.1 The population and labour force

The economy has a constant population $L = 1$ consisting of portion $q$ skilled workers and $1 - q$ unskilled workers. Consumer $i$, skilled or unskilled, has utility function

$$U_{it} = \sum_{h=t}^{\infty} G_{ih} (1 + r)^{-h+t}, \tag{1}$$

where $G$ is output consumed. It is linear such that it pins down the interest rate at $r$ for all $t$. Consumers earn wages and the profits from any licences they may hold.

## 2.2 Production

The economy has perfectly competitive producers of final output and monopolist suppliers of each of a variety of machines. In aggregate, final output can be used for consumption of final goods, to import machines for further production, or to acquire licences for new types of machine.

Using a variety expansion model (Romer, 1990), the linearly homogeneous production technology for final goods is:

$$Y_{it} = (L_{it}^{s})^{1-\alpha} \sum_{j=1}^{N_t} (X_{ijt})^{\alpha} + (AL_{it}^{u})^{1-\alpha} \sum_{j=1}^{M_t} (Z_{ijt})^{\alpha} \tag{2}$$

$Y_{it}$ is output for firm $i$ at time $t$. $L_{it}^{s}$ and $L_{it}^{u}$ are skilled and unskilled labour. $X_{ijt}$ is machine input of type $j$ used by firm $i$ at $t$. It is the quantity of each of $N$ machines (capital), which complements skilled labour. Similarly, $Z_{ijt}$ is the quantity of each of $M$ machines complementing unskilled labour. Capital depreciates fully in each period. $A < 1$ for unskilled labour makes production a function of effective units of labour, with the coefficient for skilled labour normalised to one. We will refer to $N$ and $M$ as the number of skilled machines and unskilled machines.

The production technology, which is employed in Kiley (1999), implies the elasticity of substitution between the skilled and unskilled processes is infinite. As argued in Rahman (2005), this functional form reflects the phenomenon that there are discrete different ways to produce a good. The choice of method may depend on a region's endowments.

The price of final output is unity. Firms are profit maximisers and the quantity of each type of skilled machine demanded by each firm is such that the marginal product of the machine equals its price. For the economy as a whole, we can condition demand for skilled machines on the quantity of skilled labour. Because final goods are produced using a constant returns to scale technology, we know that, in equilibrium, economy-wide demand for each skill-biased intermediate $j$ must be:

$$X_{jt} = q \left( \frac{\alpha}{P_{jt}^{X}} \right)^{\frac{1}{1-\alpha}} \tag{3}$$

$q$ is the quantity of skilled labour available to the economy. $P_{jt}^{X}$, the price of each skilled machine, is set by the firm holding the licence for that type of machine. Firms acquire this licence by importing it from abroad in exchange for exports of $Y$ at a cost described below. Firms in the technology import sector must receive ex post profits to persuade them to incur the ex ante licence cost. Once the fixed cost of acquiring the licence from abroad has been incurred, it costs 1 exported unit of $Y$, which has a price of 1, to import each machine. Using (3), the own-price elasticity of demand is $\frac{1}{1-\alpha}$ for all machines of any type. Therefore each monopolist sets a profit maximizing price of $\frac{1}{\alpha}$ for all $j, t$, so demand for each and every skilled intermediate good in the economy is equal and given by:

$$X = \alpha^{\frac{2}{1-\alpha}} q \tag{4}$$

Similarly, demand for intermediates that use unskilled labour is:

$$Z = A\alpha^{\frac{2}{1-\alpha}} (1-q) \tag{5}$$

Output for the economy is given by:

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} \left[N_t q + AM_t(1-q)\right] \tag{6}$$

### 2.3 Technology adoption

We are interested in establishing the equilibrium number of technologies $N$ and $M$ at time $t$. Most countries in the world do not develop their own technologies but acquire them from abroad. This is especially the case for developing countries.Calculations based on data from the OECD Patents database for 2003 show the top five sources[3] of patents account for 84% of the patents worldwide in the database (and 86% of OECD patents). The 16 developing countries for which data are available account for only 3%. Therefore, it is arguably more appropriate to model a developing country's technological advancement as taking place by acquisition from abroad rather than research and development.

We start by describing the decision of a potential licence holder whether or not to acquire a licence for a technology from abroad. At any time $t$, the agent considers if the value of the licence exceeds the cost. The agent would incur the cost at $t$ and start receiving profits at $t+1$. The value is the discounted present value of all future profits, so the value of any skilled licence at time $t$ is $V_t^s = \sum_i^{\infty} \left(P_{t+i}^X - 1\right) X_{t+i} \left(1+r\right)^{-i}$. Recalling $P^X = \frac{1}{\alpha}$, using (4) and defining $\Omega \equiv (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$, the per period profit from a licence for a skilled machine is $\Omega q_t$. Summing over infinite periods when the skill composition is constant, the present value of a skilled licence is:

$$V^s = \Omega \frac{q}{r} \tag{7}$$

Similarly:

$$V^u = A\Omega \frac{1-q}{r} \tag{8}$$

Equations (7) and (8) capture the market size arguments of Schmookler (1966): the value of acquiring a particular type of technology is directly proportional to the size of the market, which is determined by the quantity of workers available to work with the machine.

The specification of costs is an important feature of the model. The cost of acquiring a licence for a new technology variety depends on how many technology varieties of a particular type - skilled or unskilled - exist in the economy relative to the stock of internationally developed technologies of that type. If an economy is relatively far from the technology frontier, then it can acquire a licence for a machine that is relatively old and hence relatively cheap (but equally productive). Analagous to Kiley (1999), the cost of acquiring a licence to import a skilled machine depends on the number of machines

[3] United States, Japan, Germany, France and the United Kingdom

already in use ($N$) relative to the number available in the world ($R^s$):

$$C^s_t = \left(\beta^s \frac{N_t}{R^s_t}\right)^\kappa \text{ if } \frac{N_t}{R^s_t} < 1, \text{ where } \kappa > 0 \tag{9a}$$

$$C^s_t = \infty \text{ if } \frac{N_t}{R^s_t} \geq 1 \tag{9b}$$

The cost of acquiring from abroad a licence for an unskilled machine is given by:

$$C^u_t = \left(\beta^u \frac{M_t}{R^u_t}\right)^\kappa \text{ if } \frac{M_t}{R^u_t} < 1, \text{ where } \kappa > 0 \tag{10a}$$

$$C^u_t = \infty \text{ if } \frac{M_t}{R^u_t} \geq 1 \tag{10b}$$

$R^s_t$ is the number of skilled machine varieties available in the world and $R^u_t$ is the number of unskilled varieties. $\kappa$ is the elasticity of cost of acquiring a technology from abroad with respect to the proportional distance from the frontier.[4] Should a country be on the frontier for a type of technology, then it is of course infinitely expensive to copy a newer technology, but we expect developing countries to be well within this frontier. $\beta^s$ and $\beta^u$ allow for possible differences in the ability to adopt technologies across developing countries and between skilled and unskilled technologies. Factors affecting these parameters might be the regulatory environment or proximity to technological leaders.

The stock of skilled and unskilled technologies available worldwide is assumed to evolve exogenously according to:

$$R^s_{t+1} = \gamma^s R^s_t \tag{11}$$

$$R^u_{t+1} = \gamma^u R^u_t \tag{12}$$

The treatment of basic research as exogenous is appropriate here. A developing country is unable to influence the decisions by first world producers to develop new technologies. $\gamma^s > \gamma^u > 1$ denotes exogenous skill-biased technological change in the North (NSBTC). We assume this in the model, consistent with the empirical observations in the introduction.

By free entry, the value of acquiring a licence can never exceed the cost, so

$$V^s_t \leq C^s_t \tag{13}$$

and

$$V^u_t \leq C^u_t \tag{14}$$

In equilibrium, (13) and (14) hold with equality. Define $V = \frac{V^s}{V^u}$ as the relative value of skilled technologies, $C_t \equiv \frac{C^s}{C^u}$ as the relative cost, $\frac{N_t}{M_t} \equiv T_t$ as the ratio of skilled to unskilled technologies in the developing country, $\frac{R^s_t}{R^u_t} \equiv R_t$ as the skill bias of the world technology frontier, $\beta \equiv \frac{\beta^s}{\beta^u}$ and $\left(\frac{q}{A(1-q)}\right) \equiv Q$ as the effective ratio of skilled to unskilled labour. To summarise:

**Proposition 1** *The ratio of skilled to unskilled technologies is increasing in the relative values of skilled technologies and decreasing in the relative costs of skilled technologies:*

$$T_t = \frac{R_t}{\beta} \left(V_t\right)^{1/\kappa} \tag{15}$$

[4] A higher $\kappa$ denotes costs rise by more as the ratio of the number of machines already in use relative to those available in the world increases.

**Proof.** The result follows when (13) and (14) hold with equality and when we substitute from (9a) and (10a). ∎

(15) expresses the ratio of the technologies in terms of relative costs and relative values. This form is general and will carry over to the imperfect substitution and non-constant population cases. For our perfect substitutes production technology, in equilibrium,

$$N_t = \frac{R_t^s}{\beta^s}\left(\frac{\Omega q}{r}\right)^{1/\kappa} \tag{16}$$

$$M_t = \frac{R_t^u}{\beta^u}\left(\frac{A\Omega(1-q)}{r}\right)^{1/\kappa} \tag{17}$$

We can use (7) and (8) to show the relative values are given by the relative effective ratio of skilled to unskilled labour:

$$V = Q \tag{18}$$

Therefore, applying (15) to the production technology (2) and a constant population, we can make the following statement:

**Corollary 2** *When skilled and unskilled labour are perfect substitutes, the ratio of skilled to unskilled technologies is proportional to the relative effective skill supply ($Q$) and increasing in the skill-bias of the world technology frontier ($R$):*

$$T_t = \frac{R_t}{\beta}(Q)^{1/\kappa} \tag{19}$$

## 2.4 Wages

This section establishes the link between wages, the world technology frontier and the domestic skill supply. Labour is exogenously supplied to the economy. Individual firms hire each labour unit such that its marginal product equals the wage. For equilibrium in the economy, wages are such that, for a given level of technology and intermediates,

$$W_t^s = (1-\alpha)\,N_t\left(\frac{X}{q}\right)^{\alpha} \tag{20}$$

$$W_t^u = A^{1-\alpha}(1-\alpha)\,M_t\left(\frac{Z}{1-q}\right)^{\alpha} \tag{21}$$

Using (4) and (5), we find,

$$W_t^s = (1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}N_t \tag{22}$$

$$W_t^u = A\,(1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}M_t \tag{23}$$

(22) and (23) expose one of the simplifications induced by the production function. The terms in labour and intermediates cancel, so we see that wages are proportional to the number of technology varieties. By (16) and (17), we see this translates into a positive relationship between wages and labour supply and between wages and the research frontier:

$$W_t^s = \theta^s\frac{R_t^s}{\beta^s}\left(\frac{q}{r}\right)^{1/\kappa} \tag{24}$$

$$W_t^u = \theta^u\frac{R_t^u}{\beta^u}\left(\frac{1-q}{r}\right)^{1/\kappa} \tag{25}$$

where $\theta^s \equiv \left[(1-\alpha)^{1+\kappa}\alpha^{\left(\frac{1+\alpha+2\alpha\kappa}{1-\alpha}\right)}\right]^{1/\kappa}$ and $\theta^u \equiv \left[A^{1+\kappa}(1-\alpha)^{1+\kappa}\alpha^{\left(\frac{1+\alpha+2\alpha k}{1-\alpha}\right)}\right]^{1/\kappa}$. Letting $W_t \equiv \frac{W_t^s}{W_t^u}$ be the skill premium or degree of wage inequality, we can summarize the discussion in the following proposition:

**Proposition 3** *When skilled and unskilled labour are perfect substitutes, the skill premium is proportional to the effective relative skill supply and increasing in the skill-bias of the world technology frontier.*

$$W_t = \frac{R_t}{\beta}\frac{(Q)^{1/\kappa}}{A} \tag{26}$$

**Proof.** Dividing (24) by (25) yields the result. ■

This nests the corollary to Proposition 1 in Kiley (1999).

### 2.5 Steady-state evolution of the economy

By assumption, we have a constant rate of skill-biased technological change in the North (NSBTC). $\gamma^s > \gamma^u$ such that $\frac{R_{t+1}}{R_t} > 1$. We will say the economy is in steady state whenever the population is constant. This allows us to present a key result:

**Theorem 4** *When skilled and unskilled labour are perfect substitutes, skill-biased technological change in the North is transmitted directly to SBTC in the South such that, on the steady state growth path:*

$$\frac{T_{t+1}}{T_t} = \frac{\gamma^s}{\gamma^u} \tag{27}$$

**Proof.** From (16) and (11):

$$\frac{N_{t+1}}{N_t} = \gamma^s \tag{28}$$

From (17) and (12):

$$\frac{M_{t+1}}{M_t} = \gamma^u \tag{29}$$

Recalling $T_t \equiv \frac{N_t}{M_t}$ yields (27). ■

The simple expression (27) reflects the persistent SBTC observed in developing countries and delivers one of the key messages of the paper: holding skill composition constant, product variety expansion can be skill-biased simply because technological change research is skill-biased in the North ($\gamma^s > \gamma^u$). It captures the observation that:

> "*...developing countries must be choosing from a menu of best practices that includes an ever-increasing proportion of skill-biased technologies.*" - (Berman & Machin, 2000:3)

It is easy to confirm that skilled wages grow at $\gamma^s$ and unskilled wages grow at $\gamma^u$, which allows us to make the following statement about the link between global technological patterns and wage inequality:

**Corollary 5** *When skilled and unskilled labour are perfect substitutes, skill-biased technological change in the North is transmitted directly to wage inequality in the South such that, on the steady state growth path,*

$$\frac{W_{t+1}}{W_t} = \frac{\gamma^s}{\gamma^u} \tag{30}$$

# 3 Changes in the skill composition

We analyse an unexpected rise in $q$ from $q_0$ to $q_1$ in time period $t = 0$ to see what impact this has on the varieties of technologies.[5] Thereafter we analyse the effects of expanded educational access on a chort-by-cohort basis.

## 3.1 A one-off rise in skill supply

### 3.1.1 Technology

Inspection of (7) shows $\frac{V^s_{jt}|q_1}{V_{jt}|q_0} = \frac{q_1}{q_0}$. A rise in the number of skilled workers raises demand for each available skilled machine and therefore raises the value of a skilled licence. Therefore, initially, $V^s > C^s$. The number of varieties at $t = 0$ is not affected as it takes one period to import a new technology. However, the economy can jump to the equilibrium quantity the following period. (16) shows the ratio is:[6]

$$\frac{N_t|q_1}{N_t|q_0} = \left(\frac{q_1}{q_0}\right)^{1/\kappa}, t > 0 \tag{31}$$

The ratio holds for all $t$ as variety expansion continues at rate $\gamma^s$. Similarly, $\frac{V^u_{jt}|(1-q_1)}{V^u_{jt}|(1-q_0)} = \frac{1-q_1}{1-q_0}$ and the *equilibrium* value of unskilled technologies falls:

$$\frac{M_t|(1-q_1)}{M_t|(1-q_0)} = \left(\frac{1-q_1}{1-q_0}\right)^{1/\kappa}, t \geq t^* \tag{32}$$

Initially, $V^u < C^u$ but the number of varieties available cannot fall; the technologies have already been acquired. The *actual* number of unskilled intermediates remains constant until the research frontier ($R^u$) has advanced sufficiently. In other words, because the value of unskilled intermediates has fallen, the cost $C^u$ must fall sufficiently before technology adoption can resume. This occurs when $\frac{M_{t^*}|(1-q_1)}{M_0|(1-q_0)} > 1$. By (12) and (17), $(\gamma^u)^{t^*} > \left(\frac{1-q_1}{1-q_0}\right)^{1/\kappa}$ so that:

$$t^* > \frac{\ln \frac{1-q_0}{1-q_1}}{\kappa \ln \gamma^u} \tag{33}$$

The time required is shorter if the population change is smaller and the unskilled research frontier advances faster.[7] At $t^*$, technology imports resume at rate $\gamma^u$.

### 3.1.2 Wages

At $t = 0$, the quantity of machines and variety of technologies is unaffected by the change in skill composition. Holding the quantity of machines demanded and technology adoption constant, (20) shows skilled wages are lower than they would have been, in accordance with downward sloping labour demand. Similarly, (21) shows unskilled wages are higher than they would have been. We thus see a fall in wage

[5] An economy can experience a relatively fast rise in the skill composition if race or ethnicity barriers to highly skilled jobs are removed. In South Africa for example, people who may have been skilled were effectively barred from participating in the skilled labour force. This form of discrimination was removed, resulting in a relatively fast rise in the skill composition of labour supply.

[6] The negative effect of $\kappa$ can be explained as follows: to restore equilibrium, the rise in value generated by higher $q$ must be matched by a rise in cost. The higher the value of $\kappa$, the faster cost rises as $N_t$ moves closer to the technology frontier and thus the smaller the rise in $N_t$ needed to restore the equivalence.

[7] A large value of $\kappa$ means the cost of a new technology decreases fast as the economy falls further behind the advancing frontier.

inequality at $t_0$. However, machine demand ($X_j$ and $Z_j$) can adjust to the new skill composition for $t > 0$. (22) and (23) show that, still holding the number of varieties constant, the effect of the change in machine demand on wages exactly cancels the effect of the change in labour supply. Wage behaviour is thus driven solely by the nature of technology adoption in the perfect substitutes case.

Because the variety of unskilled intermediates remains constant, unskilled wages also remain constant until $t^*$. Thereafter, they rise at rate $\gamma^u$. Therefore, relative to what they would have earned had the change in skill supply not taken place, unskilled workers suffer a loss of $\left(\frac{1-q_1}{1-q_0}\right)^{1/\kappa}$ in all time periods ($t > 0$). However, because the number of varieties does not fall, unskilled wages never fall below the level in the period immediately preceding the rise in skill supply.

**Corollary 6** *If skilled and unskilled labour are perfect substitutes, a one-off rise in skill supply will cause unskilled workers to earn less than they would have throughout the steady-state growth path, but they do not experience an actual drop in wages.*

Similarly, skilled wages jump when the population rises before resuming their normal rate of increase $\gamma^s$. Wage inequality jumps by $\left(\frac{q_1}{q_0}\right)^{1/\kappa}$ at $t = 1$ and continues to grow at $\gamma^s$ until $t < t^*$. Once adoption of unskilled technologies resumes, wage inequality grows at $\frac{\gamma^s}{\gamma^u}$ as before. The net effect of skill supply on wage inequality is a levels effect:

$$\frac{W|q_1}{W|q_0} = \left(\frac{q_1(1-q_0)}{q_0(1-q_1)}\right)^{1/\kappa}, t \geq t^* \tag{34}$$

The results presented are so far consistent with Proposition 2 and its corollary in Kiley (1999). One-off changes in population cause annual rises in wage inequality only as long as $M_t$ is stagnant.

## 3.2 An improvement in educational access

### 3.2.1 Demography

Any improvement in the quantity and quality of education will raise the skill composition of school-age cohorts rather than an economy as a whole. Rather than experiencing an instantaneous shift, an economy's skill composition can rise only gradually as cohorts of people dying are replaced by those born and going to school.

To model this, we assume people can exist in one of three states. They can either be skilled, unskilled or deceased. The labour force (people that are not deceased) is fully employed and normalised to one such that proportion $q_t$ of the labour force is skilled and $1 - q_t$ is unskilled. In each period, both skilled and unskilled workers have probability $\delta$ of entering the deceased state (dying). In turn, proportion $\Psi$ of those in the deceased state are reborn as skilled people and $1 - \Psi$ are reborn as unskilled people. One alternative interpretation is that, upon dying at the end of one period, people are immediately reborn and exist as children at the start of the next period. Proportion $\Psi$ of children receive schooling to a given level before entering the workforce. The dynamics are captured by the following Markov process:

$$\begin{pmatrix} q_{t+1} \\ 1-q_{t+1} \\ D_{t+1} \end{pmatrix} = \begin{pmatrix} 1-\delta & 0 & \Psi \\ 0 & 1-\delta & 1-\Psi \\ \delta & \delta & 0 \end{pmatrix} \begin{pmatrix} q_t \\ 1-q_t \\ D_t \end{pmatrix} \tag{35}$$

The solution to the system is:

$$\begin{pmatrix} q_{t+1} \\ 1-q_{t+1} \\ D_{t+1} \end{pmatrix} = \begin{pmatrix} \Psi \\ 1-\Psi \\ \delta \end{pmatrix} + (\Psi - q_0) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} (1-\delta)^t \tag{36}$$

$q_0$ is the proportion of skilled workers at the initial date $t = 0$. At any given time, the number of deceased people is constant at $\delta$; all $\delta$ people are reborn to replace the $\delta$ who die, maintaining a constant living population.

If the proportion of the new cohort being educated $\Psi$ is the same as the proportion of the labour force that is skilled, then the proportion of skilled workers remains constant at $\Psi$; that is, $q = \Psi$. If not, we see that the proportion of skilled workers approaches its steady state $\Psi$ as $t$ gets large. In particular:

$$\frac{q_{t+1}}{q_t} = 1 + \left(\frac{\Psi - q_t}{q_t}\right)\delta \tag{37a}$$

$$\frac{1-q_{t+1}}{1-q_t} = 1 - \left(\frac{\Psi - q_t}{1-q_t}\right)\delta \tag{37b}$$

Because each consumer's lifespan is uncertain, agents may leave unexpected and unintended bequests or debts. Therefore, building on Blanchard (1985) and Yaari (1965), each consumer takes a bet offered by insurance companies. If the consumer dies during the time period, she gives up all her assets. If she remains alive, she receives a certain portion of her assets. Insurance companies offer this risklessly and without profit. The actuarially fair portion they pay out contingent on the consumer staying alive is $\frac{\delta}{1-\delta}$. In other words, insurance companies collect the assets from the $\delta$ people dying every year and turn proportion $\frac{\delta}{1-\delta}$ of these assets over to the $1 - \delta$ people who stay alive.

We now model the effects of a new education policy that gradually raises the proportion of skilled workers in the economy. We implicitly assume the main driver of educational attainment is ease of access through the supply of education, not shifts in demand by individuals. This assumption is appropriate in the context of widening access to formerly barred segments of the population.

We assume the economy initially has skill proportion $q_0$, with the number of people being educated $\Psi = \Psi_0$ so that the proportion is constant. At $t = 0$ ($t_0$), better access to education raises $\Psi$ to $\Psi_1 > \Psi_0$. This is credibly announced at $t_0$ but only starts to take effect one period later as given by (36). Over time, the economy's proportion of skilled workers will move towards $\Psi_1$. These demographics are known to all agents in the economy.

### 3.2.2 Technology adoption

When $N$ and $M$ are at their equilibrium values, but the skill composition is changing, we refer to this as the equilibrium transition path. However, it is possible for the equilibrium level of $M$ to be below its actual level, such that the acquisition of unskilled machines ceases. This is a period when the economy is not on the equilibrium transition path. We start by describing the paths of $N$ and $M$ on the equilibrium transition path. Thereafter, we present a fuller description of the paths of the values and costs of skilled and unskilled licences, including the equilibrium transition path and periods when unskilled technology acquisition is suspended.

**Technology along the equilibrium transition path** The cost of acquiring a licence is as in (9a) and (10a). Similarly, the value of a licence is the discounted present value of the profits from selling machines. The per period profit from a licence is $\Omega q_t$ for a skilled machine, so the value of a skilled licence is:

$$V_t^s = \Omega \left[\sum_{i=1}^{\infty} \frac{q_{t+i}}{(1+r)^i}\right] \tag{38}$$

Because $q_t$ is changing, we use (36) to find, for a skilled licence,

$$V_t^s = \Omega \left[ \frac{\Psi_1}{r} - \frac{(\Psi_1 - \Psi_0)(1-\delta)^{t+1}}{r+\delta} \right] \tag{39}$$

Similarly, for an unskilled licence,

$$V_t^u = A\Omega \left[ \frac{1-\Psi_1}{r} + \frac{(\Psi_1 - \Psi_0)(1-\delta)^{t+1}}{r+\delta} \right] \tag{40}$$

The equilibrium varieties of technologies are those that equate their value and cost:

$$N_t = \frac{R_t^s}{\beta^s} \Omega^{1/\kappa} \left[ \frac{\Psi_1}{r} - \frac{(\Psi_1 - \Psi_0)(1-\delta)^t}{r+\delta} \right]^{1/\kappa} \tag{41}$$

$$M_t = \frac{R_t^u}{\beta^u} (A\Omega)^{1/\kappa} \left[ \frac{1-\Psi_1}{r} + \frac{(\Psi_1 - \Psi_0)(1-\delta)^t}{r+\delta} \right]^{1/\kappa} \tag{42}$$

Recall $\Psi_0 = q_0$ is the original proportion of skilled workers and $\Psi_1$ is the new proportion of people being educated and the eventual new steady state proportion of skilled workers. As $t$ gets large, the terms converge on the present value of a constant population, as in (7) and (8). We can make the following statement about the growth rates of the varieties on the equilibrium transition path.

**Theorem 7** *If skilled and unskilled labour are perfect substitutes, improved educational access in a developing country will result in accelerated SBTC such that, on the equilibrium transition path,*

$$\frac{T_{t+1}}{T_t} > \frac{\gamma^s}{\gamma^u} \tag{43}$$

**Proof.** The growth rate of $N$ is:

$$\frac{N_{t+1}}{N_t} = \gamma^s \left[ 1 + \frac{\Psi - q_t}{q_t} \delta \right]^{1/\kappa} \text{ (by (16) and (37a))} \tag{44}$$

$$= \gamma^s \left[ 1 + \phi_t^s \right]^{1/\kappa} > \gamma^s \text{ for finite } t \text{ (by (41))} \tag{45}$$

Here, $\phi_t^s = \frac{r\delta(\Psi_1 - \Psi_0)(1-\delta)^t}{\Psi_1(r+\delta) - r(\Psi_1 - \Psi_0)(1-\delta)^t} > 0$ for finite values of $t$. Also,

$$\frac{M_{t+1}}{M_t} = \gamma^u \left[ 1 + \frac{\Psi - q_t}{q_t} \delta \right]^{1/\kappa} \text{ (by (17) and (37b))} \tag{46}$$

$$= \gamma^u \left[ 1 - \phi_t^u \right]^{1/\kappa} < \gamma^u \text{ for finite } t \text{ (by (42))} \tag{47}$$

Here, $\phi_t^u = \frac{r\delta(\Psi_1 - \Psi_0)(1-\delta)^t}{(1-\Psi_1)(r+\delta) - r(\Psi_1 - \Psi_0)(1-\delta)^t} > 0$ for finite values of $t$. (43) follows easily from combining (45) and (47). ■

**Corollary 8** *On the equilibrium transition path, the rate of SBTC will fall back towards the steady-state rate of* $\frac{\gamma^s}{\gamma^u}$.

$\underset{t\to\infty}{Lim}\phi_t^s = 0$ and $\underset{t\to\infty}{Lim}\phi_t^u = 0$ and hence $\underset{t\to\infty}{Lim}\frac{T_{t+1}}{T_t} = \frac{\gamma^s}{\gamma^u}$.

It is clear from (45) that growth in skilled technologies is accelerated by the rising proportion of skilled workers. This combines the effects of a fall in cost from an advancing research frontier with the increasing attractiveness of a rising skill composition. As $t$ gets large and the population approaches its steady state, technology growth falls to the steady state rate $\gamma^s$. Similarly, (47) shows that growth is less than the steady state value: falling costs are counteracted by the decreasing attractiveness of the market for unskilled technologies. As $t$ gets large, the growth rate rises back to $\gamma^u$.

**The path of $V_t^s$ and $C_t^s$** (39) can be used to compare the values of a skilled licence with and without the change in educational access. Noting that $V_t^s|\Psi_0 = V_0^s|\Psi_0$ for all $t$ because the skill proportion would have remained constant,

$$\frac{V_t^s|\Psi_1}{V_t^s|\Psi_0} = \frac{\Psi_1}{\Psi_0} - \frac{r}{r+\delta}\left(\frac{\Psi_1}{\Psi_0} - 1\right)(1-\delta)^t \tag{48}$$

$\lim_{t\to\infty} \frac{V_t^s|\Psi_1}{V_t^s|\Psi_0} = \frac{\Psi_1}{\Psi_0}$ so the value eventually changes by the same amount as reported for the one-off change. To characterise what happens initially, we set $t = 0$. In this case $\frac{V_0^s|\Psi_1}{V_0^s|\Psi_0} = \frac{\Psi_1}{\Psi_0} + \frac{r}{r+\delta}\left(1 - \frac{\Psi_1}{\Psi_0}\right)$: the initial jump in value is bigger if $\delta/r$ is bigger. If we set $\delta = r$, the initial change is approximately half of the eventual change. After the initial jump, the value continues to rise in small increments towards its steady state value. Figure 1 shows the approximate initial jump and path of $V^s = C^s$ as well as their new steady state values. Using (41), the level of skilled licences in period one relative to what it would have been is given by $\frac{N_1|\Psi_1}{N_1|\Psi_0} = \left[\frac{\Psi_1}{\Psi_0} - \frac{r}{r+\delta}\left(\frac{\Psi_1}{\Psi_0} - 1\right)(1-\delta)\right]^{1/\kappa} > 1$. Dropping all terms in $r\delta$ because they are second order relative to the magnitude of the initial jump and assuming $\delta = r$, $\frac{N_1}{N_0} \approx \left[\frac{\Psi_0+\Psi_1}{2\Psi_0}\right]^{1/\kappa}$. After the initial jump, $N$ rises along the equilibrium transition path as given in (44). This rate is gradually decreasing over time and approaches $\gamma^s$.

**The path of $V_t^u$ and $C_t^u$** The change in the value of unskilled licences is similarly:

$$\frac{V_t^u|\Psi_1}{V_t^u|\Psi_0} = \frac{1-\Psi_1}{1-\Psi_0} + \frac{r}{r+\delta}\left(1 - \frac{1-\Psi_1}{1-\Psi_0}\right)(1-\delta)^t \tag{49}$$

The initial jump at $t = 0$ is $\frac{V_0^u|\Psi_1}{V_0^u|\Psi_0} = \left(\frac{1-\Psi_1}{1-\Psi_0}\right) + \frac{r}{r+\delta}\left(\frac{1-\Psi_1}{1-\Psi_0}\right)$. If $\delta = r$, the downward jump is half the size of the steady state change. Therefore, it is likely that the equilibrium value of $M$ - the value of $M$ such that $V_t^u = C_t^u$ - will be less than the actual value. $M$ therefore remains constant until $t^+$, when $(\gamma^u)^{\kappa t^+} > \left(\frac{V_{t^+}^u|\Psi_1}{V_0|\Psi_0}\right)$.

To understand the significance of $t^+$ note that the research frontier advances to reduce the cost of acquiring a new technology. However, the value of a licence also declines as the population falls. At first, whether the cost or value fall quicker depends on the parameters. Over time, the rate of fall in value decelerates to 0 while the research frontier keeps advancing and cost keeps falling. Eventually, cost will fall faster. Technology adoption resumes once the fall in cost has caught up with the fall in value. $t^+$ denotes this period:

$$t^+ : t > \frac{\ln(1-\Psi_0)(r+\delta) - \ln\left[(1-\Psi_1)(r+\delta) + r(\Psi_1 - \Psi_0)(1-\delta)^t\right]}{\kappa \ln \gamma^u} \tag{50}$$

$t$ appears on both sides of the inequality. To characterise the solution for $t^+$, note that the total fall in value is given by the steady state fall, such that $t^*$ given by (33) is an upper bound. Similarly, the initial jump provides an approximate lower bound for $t^+$. So, for example, for a rise in the skill proportion from 20% to 40% with $\kappa = 1$ and $\gamma^u = 1.02$, the upper bound is 15 years and the approximate ($\delta = r$) lower bound is 7 years. Figure 2 illustrates the actual (separate) paths of $V^u$ and $C^u$, the steady state values, $t^+$ (when imports of unskilled licences resume) and $t^*$ (the upper bound for $t^+$).

Once technology imports resume, the rate of technology adoption along the equilibrium transition path

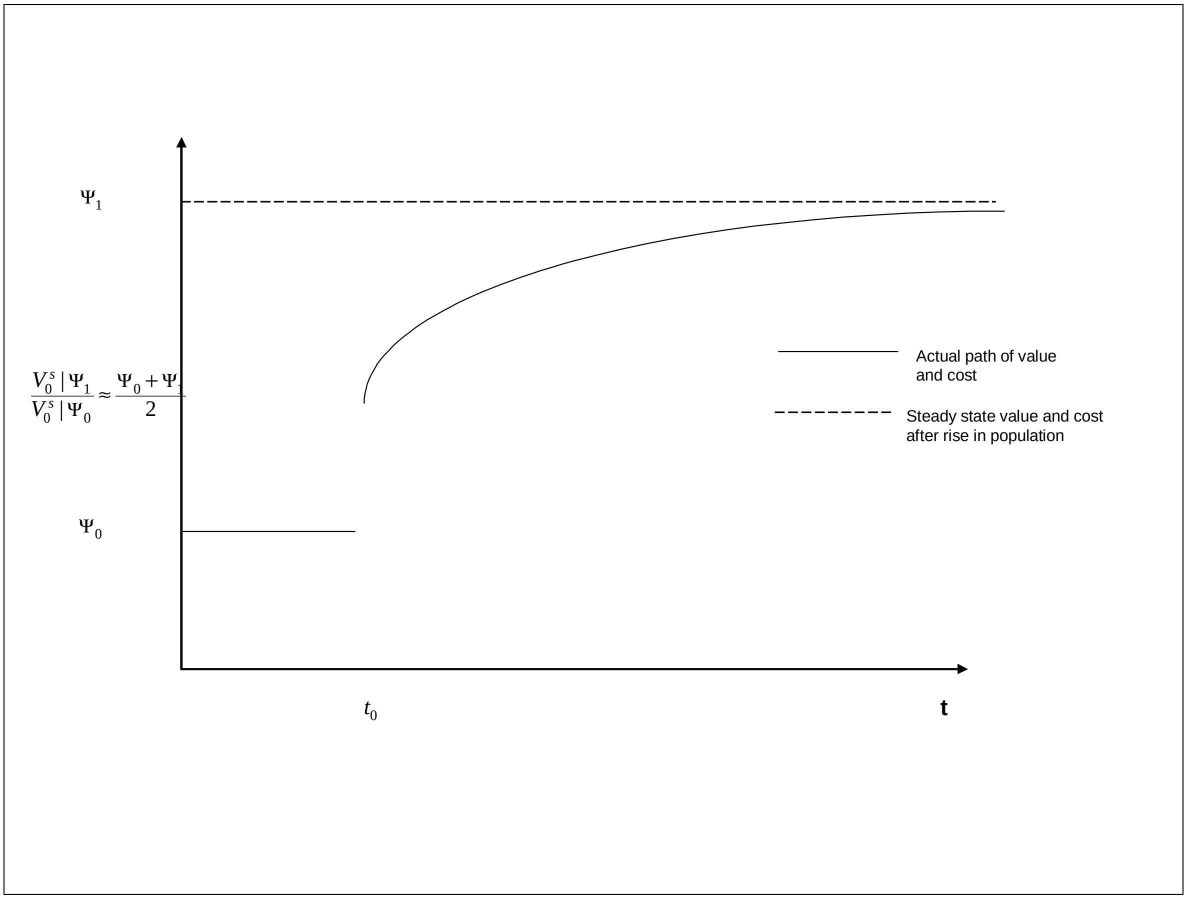


Figure 1: The evolution of $V^s$ and $C^s$ after expanded educational access

is given by (47). That expression shows that the rate of technology adoption is below $\gamma^u$ but accelerates until it reaches the rate of frontier growth. These results echo Proposition 2 in Kiley (1999) but the stagnation is now driven by a continually falling value of unskilled technologies rather than a one-off fall.

### 3.2.3 Wages

Growth in wages has exactly the same pattern as growth in technologies.

**Corollary 9** *While the skill composition of the labour force is rising, wage inequality is accelerated along the equilibrium transition path. Once the skill composition settles at its new steady-state, wage inequality grows at the steady-state rate* $\frac{\gamma^s}{\gamma^u}$.

The effect of the initial jump in $N$ is to raise skilled wages at $t_1$ by $\frac{W_1^s|\Psi_1}{W_1^s|\Psi_0} = \left[\frac{\Psi_1}{\Psi_0} - \frac{r}{r+\delta}\left(\frac{\Psi_1}{\Psi_0} - 1\right)(1-\delta)\right]^{1/\kappa}$. For example, dropping terms in $\delta r$, setting $\kappa = 1$ and making $\delta = r$, the proportional jump in wages is approximately half the proportional change in the steady state skill composition. Skilled wages continue to grow at $\frac{W_{t+1}^s}{W_t^s} = \gamma^s \left(1 + \phi_t^s\right)^{1/\kappa}$ and decelerate to $\gamma^s$ as $q_t \to \Psi_1$.

While skilled wages jump and grow fast, unskilled wages are stuck at $W_0^u$. They remain there until $t^+$, when unskilled technology adoption resumes. From $t^+$, unskilled wages grow at $\frac{W_{t+1}^u}{W_t^u} = \gamma^u \left(1 - \phi_t^u\right)^{1/k} <$

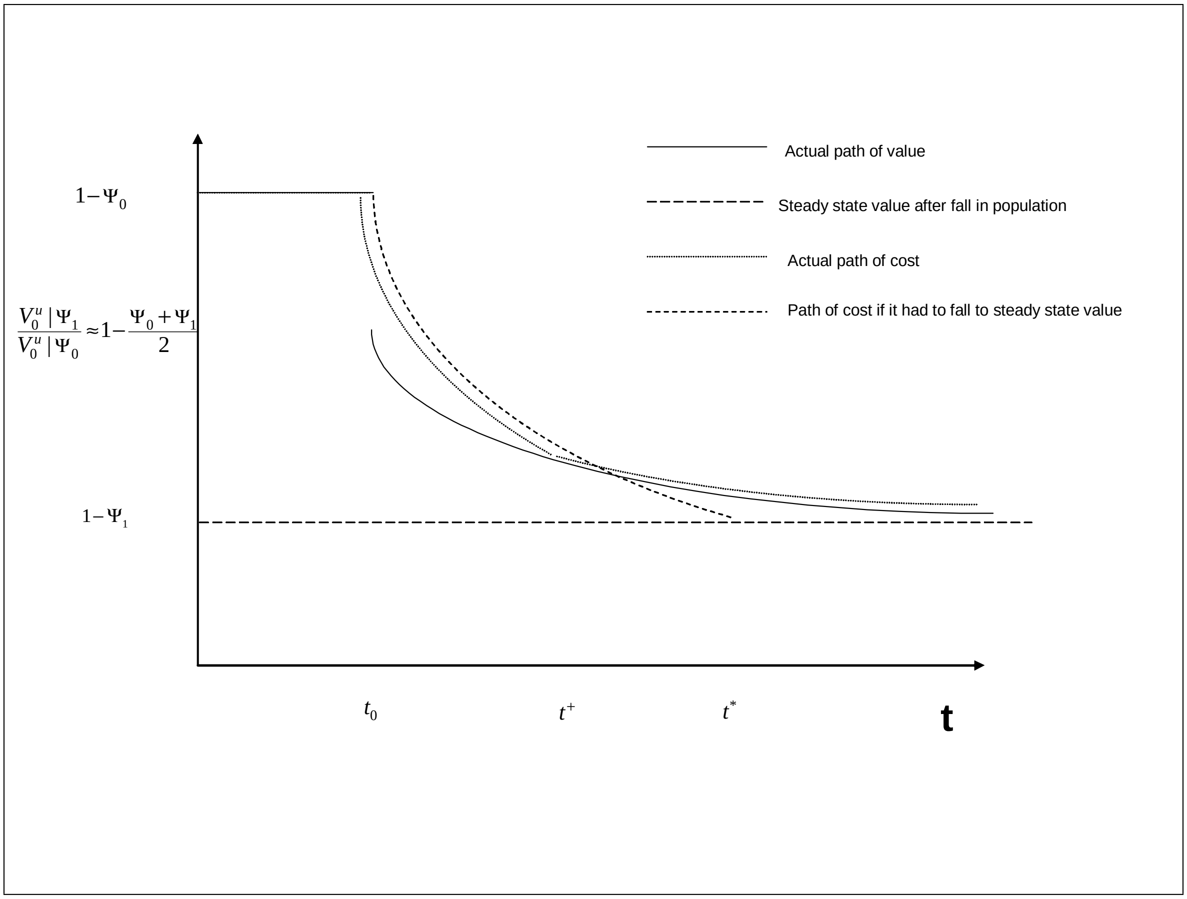


Figure 2: Evolution of $V^u$ and $C^u$ after a rise in $\Psi$ from $\Psi_0$ to $\Psi_1$. Graph scale divided by $\frac{\Omega}{Ar}$. $t^+$ denotes when marginal cost has caught up to value and imports of unskilled patents resume.

$\gamma^u$. This is a period of accelerated wage inequality:

$$\frac{W_{t+1}}{W_t} = \frac{\gamma^s}{\gamma^u}\left(\frac{1+\phi^s}{1-\phi^u}\right)^{1/\kappa} > \frac{\gamma^s}{\gamma^u}$$

We therefore have potentially long periods of accelerated wage inequality. The continual rise in wage inequality driven by differential advances in the research frontiers is compounded as the skill composition of the workforce gradually rises. Furthermore, at $t_1$, there is a one-off jump in relative wages. The comparative statics section showed that, once the skill composition settles at its new level, the effect of skill supply is a levels effect. The fact that the unskilled labour force falls for a long time makes it possible for unskilled wages to remain stagnant for long periods while skilled wages advance at a higher (although decelerating) rate.

# 4 Imperfect substitutes

Relaxing the assumption of perfect substitution between skilled an unskilled labour breaks the unequivocal relationship between skill supply and the wage premium. Using a CES technology, firms choose to make final output with a combination of imperfectly substitutable skilled and unskilled processes. This choice is in part affected by the relative prices of these processes. A rise in the supply of skilled labour and/or the number of skilled machine varieties would lead to a change in relative prices, which has a feedback effect on their marginal products. This feedback is the source of complication in the relationship between skill supply and wage premia. It also affects the relationship between the skill-bias of the technology frontier and domestic technology adoption. In this section, we describe the role of changes in relative prices and how this effects the results presented thus far. We present the conditions, in terms of the elasticity of substitution between skilled and unskilled labour, under which a rise in skill supply and/or the skill-bias of the technology frontier would lead to SBTC and increased wage inequality in the South. Finally we compare these conditions with existing empirical estimates of the elasticities.

## 4.1 Model set-up

### 4.1.1 Production

Total output of final goods is a CES aggregate of two types of intermediate:

$$Y_t = \left[ (y_t^s)^{\frac{\epsilon-1}{\epsilon}} + (y_t^u)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{51}$$

Final output is produced by perfect competitors using two intermediate inputs purchased from intermediates producers and is sold at a price of unity. $\epsilon > 0$ is the finite elasticity of substitution between intermediate inputs. Individual producers take the price of final output and intermediate input prices as given before choosing their optimal quantities of intermediates. For the economy to be in equilibrium, intermediates must have prices $p^s$ and $p^u$ such that:

$$\frac{p_t^s}{p_t^u} = \left( \frac{y_t^s}{y_t^u} \right)^{-\frac{1}{\epsilon}} \tag{52}$$

Intermediates are produced by $i$ perfectly competitive firms, where $y^s$ uses skilled labour and $N$ different machines while $y^u$ uses unskilled labour and $M$ different machines. Specifically, $y_{it}^s = (L_{it}^s)^{1-\alpha} \sum_{j=1}^{N_t} X_{ijt}^{\alpha}$ and $y_{it}^u = A(L_{it}^u)^{1-\alpha} \sum_{j=1}^{M_t} Z_{ijt}^{\alpha}$.

The labour force is as described in and around equation (1). While we will ultimately describe the equilibrium with constant skill proportions, we initially allow for time subscripts on the proportions of skilled and unskilled labour. Firms making $y^s$ take $p^s$ and factor prices as given and maximize profits. As before, the price set by the monopolist is $\frac{1}{\alpha}$ and demand for the whole economy is given by

$$X_t = \alpha^{\frac{2}{1-\alpha}} (p_t^s)^{\frac{1}{1-\alpha}} q_t \tag{53}$$

while economy-wide demand for each unskilled machine is

$$Z_t = A\alpha^{\frac{2}{1-\alpha}} (p_t^u)^{\frac{1}{1-\alpha}} (1 - q_t) \tag{54}$$

Economy-wide output of skilled and unskilled intermediates is:

$$y_t^s = N_t q_t^{1-\alpha} X_t^{\alpha} \tag{55a}$$

$$y_t^u = M_t A^{1-\alpha} (1 - q_t)^{1-\alpha} Z_t^{\alpha} \tag{55b}$$

### 4.1.2 Prices and wages

An exogenous change in the relative skill supply would lead to a rise in the relative quantity of $y^s$ produced. By (52), this would necessitate a relative price adjustment. If the ratio of $N$ to $M$ were to change, this would also necessitate a relative price adjustment. Using (52) and (55),

$$p \equiv \left(\frac{p^s}{p^u}\right)^{\frac{1}{1-\alpha}} = \left(\frac{Nq}{MA(1-q)}\right)^{\frac{-1}{\sigma}} = (TQ)^{\frac{-1}{\sigma}}\ , \tag{56}$$

where $\sigma = \epsilon + \alpha - \epsilon\alpha$ is the elasticity of substitution between skilled and unskilled labour. It captures the percentage change in relative demand for the two factors due to the change in relative factor prices at constant output; that is $-\frac{d\log\frac{L^s}{L^u}}{d\log\frac{w^s}{w^u}}$. There is a negative relationship between the relative price of the skill intensive good on the one hand and the relative number of skilled technologies on the other. We can show $\epsilon = 1 \Leftrightarrow \sigma = 1$ and $d\sigma/d\epsilon > 0$. Also, $\epsilon \to \infty \Leftrightarrow \sigma \to \infty$. Therefore, as $\epsilon \to \infty$ , changes in relative labour quantities or relative numbers of machine varieties would have no effect on $p$. For finite $\epsilon$, (56) has a major effect on the decision to acquire a licence, as will be explained in section 4.1.3.

Producers of intermediate goods hire labour such that wage equals marginal revenue product. For equilibrium in the economy, relative wages are given by:

$$W \equiv \frac{W^s}{W^u} = \frac{N}{AM}p = \left(\frac{N}{AM}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{q}{1-q}\right)^{\frac{-1}{\sigma}} \tag{57}$$

(57) mirrors the findings of Acemoglu (2002a). The far right of the equation shows relative wages are affected by two things. First, the standard substitution effect, where a rise in the relative quantity of skilled labour reduces the relative skilled wage. Second, relative technologies. The effect of relative technologies can be positive or negative. As will be explained shortly, a rise in $N/M$ raises the relative physical productivity of skilled labour but, via its effect on relative product prices, can result in a net positive or negative effect on the relative marginal revenue products. In the simpler perfect substitutes setup, the only impact is on marginal physical product so the effect is always positive. Equation (57) describes the direct relationship between wages and the skill supply. It also describes the relationship between wages and technology. The next section describes how technology adoption is determined by the skill supply and by international technology patterns.

### 4.1.3 Technology adoption

The marginal cost for a machine of any type is unity and the cost of acquiring a licence is as given in (9a) and (10a), although we set $\beta^s = \beta^u = 1$ for simplicity:

$$C_t^s = \left(\frac{N_t}{R_t^s}\right)^{\kappa} \text{ if } \frac{N_t}{R_t^s} < 1, \text{ where } \kappa > 0 \tag{58a}$$

$$C_t^s = \infty \text{ if } \frac{N_t}{R_t^s} \geq 1 \tag{58b}$$

and

$$C_t^u = \left(\frac{M_t}{R_t^u}\right)^{\kappa} \text{ if } \frac{M_t}{R_t^u} < 1, \text{ where } \kappa > 0 \tag{59a}$$

$$C_t^u = \infty \text{ if } \frac{M_t}{R_t^u} \geq 1 \tag{59b}$$

Much of the discussion will set the cost elasticity $\kappa = 1$, indicating when this is the case. For $\frac{N_t}{R_t^s} < 1$ and $\frac{M_t}{R_t^u} < 1$, the relative cost is:

$$C_t = \left(\frac{T_t}{R_t}\right)^{\kappa} \tag{60}$$

Profit from the sales of skilled and unskilled machines in a given period is:

$$\pi^x = \Omega\left(p_t^s\right)^{\frac{1}{1-\alpha}} q_t \tag{61a}$$

$$\pi^z = \Omega\left(p_t^u\right)^{\frac{1}{1-\alpha}} A(1-q_t) \tag{61b}$$

We can use (61a) to show that the value of a skill-augmenting licence is:

$$V_t^s = \Omega\left[\sum_{i=1}^{\infty} \frac{q_{t+i}\left(p_{t+i}^s\right)^{\frac{1}{1-\alpha}}}{(1+r)^i}\right] \tag{62}$$

The value of acquiring a licence today is affected by the future path of prices, which in turn is affected by the future path of $T$ (cf. (56)). Analogously:

$$V_t^u = A\Omega\left[\sum_{i=1}^{\infty} \frac{(1-q_{t+i})\left(p_{t+i}^u\right)^{\frac{1}{1-\alpha}}}{(1+r)^i}\right] \tag{63}$$

The ratio of values is:

$$V_t = \frac{\left[\sum_{i=1}^{\infty} \frac{q_{t+i}\left(p_{t+i}^s\right)^{\frac{1}{1-\alpha}}}{(1+r)^i}\right]}{A\left[\sum_{i=1}^{\infty} \frac{(1-q_{t+i})\left(p_{t+i}^u\right)^{\frac{1}{1-\alpha}}}{(1+r)^i}\right]} \tag{64}$$

While prospective agents consider the values of skilled and unskilled licences independently, they rely on the paths of prices $p_t^s$ and $p_t^u$. To bring out the effects of relaxing the assumption of perfect substitution between skilled and unskilled labour as simply as possible, we will from now on assume constant values of $Q \equiv \frac{q}{A(1-q)}$ and $R \equiv \frac{R^s}{R^u}$. This will allow us to find an equilibrium in which $p$, $V$ and $T \equiv \frac{N}{M}$ are constant. Under these assumptions, equation (64) can be simplified to:

$$V = Q^{\frac{\sigma-1}{\sigma}} T^{\frac{-1}{\sigma}} \tag{65}$$

A rise in $Q$ raises the relative marginal (physical) product of skilled machines. For finite elasticities of substitution, there is also a negative effect, because the rise in $Q$ raises $y^s/y^u$ and thus lowers $p$. This lowers the relative marginal revenue product of each machine. $\sigma$ determines the size of the negative effect and hence whether the overall effect is positive ($\sigma > 1$) or negative ($\sigma < 1$), as shown in (65). Equation (65) also shows that the relative supply of skilled machines has a negative effect. This is (also) because of an effect on $p$ and hence the relative marginal revenue products of $X$ and $Z$.

In equilibrium, the relative cost of acquiring a skilled licence must equal the relative value:

$$V_t = C_t \tag{66}$$

We can substitute from (60) to confirm proposition 1 still holds: with $\beta = 1$, $T_t = R_t\left(V_t\right)^{1/\kappa}$. We can also adapt our corollary from the perfect substitutes case for a constant $T$:

**Corollary 10** *When skilled and unskilled labour are imperfect substitutes and $R, Q$ are constant over* *time, the ratio of skilled to unskilled technologies ($T$) is increasing in the relative effective skill supply ($Q$) if and only if $\sigma > 1$ but is always increasing in the skill-bias of the world technology frontier ($R$):*

$$T = Q^{\frac{\sigma-1}{\sigma k+1}} R^{\frac{\sigma k}{\sigma k+1}}, k > 0 \quad (67)$$

$$T = Q^{\frac{\sigma-1}{\sigma+1}} R^{\frac{\sigma}{\sigma+1}}, k = 1 \quad (68)$$

This is shown by substitution from (60) and (65). We will use this result as well as (57) to conduct comparative statics on wages. Before we do that, we generalise the treatment from the perfect substitutes case and rephrase the discussion of (64) and (65), which is in terms of factor quantities and product prices, in terms of factor shares.

**Proposition 11** *When skilled and unskilled labour are imperfect substitutes and $R, Q$ are constant over time, the value of a licence for a skilled machine relative to that for an unskilled machine is directly proportional to the share of skilled labour in output relative to that of unskilled labour.*

**Proof.** Using the fact that wage equals marginal revenue product and that $T, Q$ are constant, we can write (64) as $V = \frac{qw^s}{(1-q)w^u}\frac{1}{T}$. But $qw^s/(1-q)w^u \equiv \zeta$ is an expression for the relative factor shares of skilled and unskilled labour, so,

$$V = \frac{\zeta}{T} \quad (69)$$

■

Equation (69) expresses the relative value of skilled and unskilled technologies in terms of relative factor shares and relative technology availability. Factor shares capture both the induced innovations (Hicks, 1963) and market size (Schmookler, 1966) arguments for factor-biased technological change. Hicks (1963) introduces factor saving inventions as those which increase the marginal product of the other factor relative to that factor. Such inventions seek to economise on the use of the more expensive factor, are thus spurred by changes in relative factor prices and are called "induced innovations" (pg 125). In contrast, Schmookler (1966) argues a higher quantity of a factor raises the marginal product of machines complementing that factor and increases demand for that type of machine.

We can combine the two arguments into "market attractiveness" based on relative shares. Holding technology constant, a rise in the supply of skilled labour will raise (lower) its factor share if the elasticity of substitution is more (less) than unity. Thus, if $\sigma > 1$, a rise in relative skill supply increases the skill share and overall makes the value of adopting a skilled machine licence relatively more attractive. By free entry and using (69) and (60), this translates into a relationship between the ratio of technologies and relative factor shares:

**Corollary 12** *When skilled and unskilled labour are imperfect substitutes and $R, Q$ are constant, the ratio of skilled to unskilled technologies is positively related to the relative factor share of skilled labour and positively related to the skill-bias of the world technology frontier:*

$$T = R^{\frac{k}{1+k}} \zeta^{\frac{1}{1+k}}, k > 0 \quad (70)$$

$$T = \sqrt{R\zeta}, k = 1 \quad (71)$$

The relative factor shares of skilled and unskilled labour represent the relative attractiveness of skilled machines while $R^{-1}$ represents the relative cost of skilled machines. (70) is consistent with the empirical work of Caselli & Coleman (2000:23). As shown in figure 3, their cross country data exhibits a strong

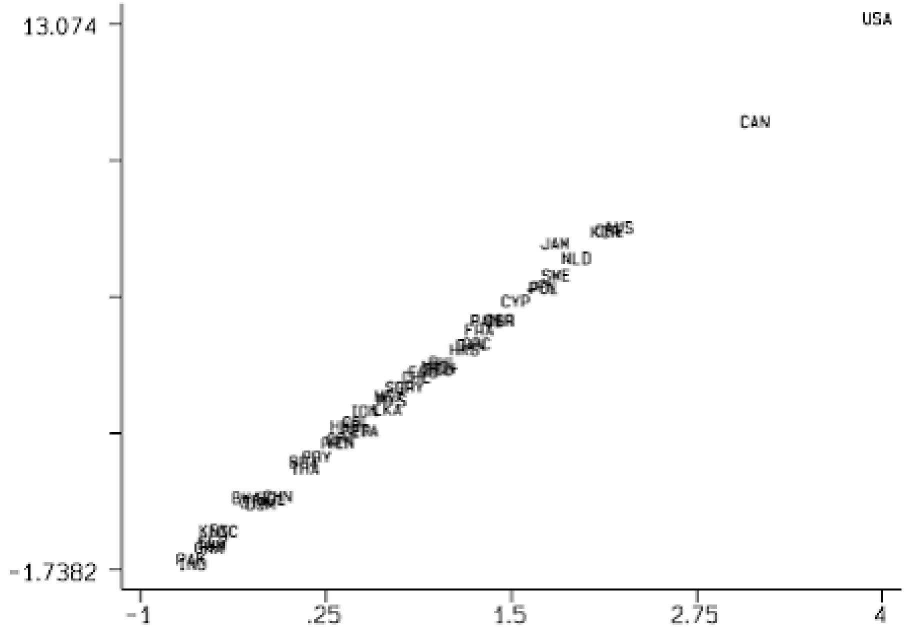


Figure 3: Scatterplot revealing relationship between relative labour efficiency (y-axis) and relative factor shares (x-axis). Source: Caselli & Coleman (2000:23, figure 9). Using their interpretation and our notation, relative labour efficiency is $\log N - \log AM$. Relative factor shares are $\log \varsigma_s - \log \varsigma_u$. Each observation is a country.

positive correlation between relative factor shares and the productivities of skilled workers relative to unskilled workers, which they interpret as the relative numbers of machines skilled workers have. They propose this is evidence of appropriate technology adoption driven by factor endowments.

Corollary 12 is useful for combining the market size and induced innovations arguments as well as for comparison with the findings of Caselli & Coleman (2000). However, for effective comparative statics, we solve explicitly for $T$ to obtain (67) or (68).

#### 4.1.4 Equilibrium in steady state

To summarise, equilibrium in the final goods ($Y$) market has all goods sold at price unity for consumption, to acquire licences or to import machines. Equilibrium in the intermediates $(y^s, y^u)$ market is established by relative price $p$ such that (cf. equation (56))

$$p = (TQ)^{\frac{-1}{\sigma}} \tag{72}$$

Supply equals demand for each of $N + M$ machine varieties sold at the monopoly price of $\frac{1}{\alpha}$. The labour market clears at the relative wage given by equation (57):

$$W = \left(\frac{N}{AM}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{q}{1-q}\right)^{\frac{-1}{\sigma}} \tag{73}$$

The relative value of acquiring a skilled technology equals the relative cost: $V_t = C_t$. From proposition 1, $T_t = R_t (V_t)^{\frac{1}{\kappa}}$. While the varieties of skilled ($N$) and unskilled ($M$) machines can grow and there is growth in $R^s$ and $R^u$, we restrict ourselves to analyses of the steady state *ratio* $T$. We therefore have

$V = Q^{\frac{\sigma-1}{\sigma}} T^{\frac{-1}{\sigma}}$ (cf. 65) and the equilibrium condition:

$$T = Q^{\frac{\sigma-1}{\sigma k+1}} R^{\frac{\sigma k}{\sigma k+1}}, k > 0 \quad (74a)$$

$$T = Q^{\frac{\sigma-1}{\sigma+1}} R^{\frac{\sigma}{\sigma+1}}, k = 1 \quad (74b)$$

## 4.2 Comparative statics

We model NSBTC as a one-off rise in $R$ and a rise in skill supply as a one-off rise in $Q$. Specifically, the comparative statics shows what elasticity parameter values are needed for NSBTC and a rise in skill supply to generate SBTC and increased wage inequality.

As introduced in corollary (10), inspection of equation (74b) shows that a rise in relative skill supply would lead to a rise in $T$ if $\sigma > 1$. The intuition is clear with the help of (71). If $\sigma > 1$, a rise in the supply of skilled labour will lead to a rise in its factor share. This is because the factor inputs are relatively easy to substitute and the wage of skilled labour, *ceteris paribus,* would not need to fall by much to accommodate the rise in skill supply. If $\sigma < 1$, the number of skilled machines relative to unskilled ones would fall after a rise in skill supply. Importantly, we can also see from equation (74b) that a rise in $R$ has a positive effect on $T$, *regardless* of $\sigma$. We now turn to the comparative static effects on wages.

**Proposition 13** *When skilled and unskilled labour are imperfect substitutes, a rise in skill supply will lead to a rise in wage inequality iff* $\sigma > 2 + \kappa$*.*

**Proof.** The derivative of (73) with respect to $Q$ is $\frac{d \log W}{d \log Q} = \frac{\partial \log W}{\partial \log T}\frac{d \log T}{d \log Q} - \frac{1}{\sigma}$, but we know from (73) that $\frac{\partial \log W}{\partial \log T} = \frac{\sigma-1}{\sigma}$. We also know from (74a) that $\frac{d \log T}{d \log Q} = \frac{\sigma-1}{\sigma\kappa+1}$. Therefore $\frac{d \log W}{d \log Q} = \frac{\sigma-2-\kappa}{\sigma\kappa+1}$. ∎

A rise in $N$ relative to $M$ has two effects on wages. First, it increases the relative (physical) productivity of skilled labour. Second, it increases $y^s/y^u$, which necessitates a fall in $p$ and therefore has a negative effect on the relative marginal revenue product of skilled labour. The net effect on relative wages depends on $\sigma$. If $\sigma > 1$, a rise in $T$ will have a positive effect on the skill premium, because the second effect is relatively small. Conversely, $\sigma < 1$ means relative wages will have to adjust a lot so that the overall effect is negative. Building on this, we see that:

$$\frac{d \log W}{d \log Q} = \frac{\partial \log W}{\partial \log T}\frac{d \log T}{d \log Q} - \frac{1}{\sigma} \quad (75)$$

$$= \underbrace{\frac{\sigma-1}{\sigma}\frac{\sigma-1}{\sigma k+1}}_{\text{directed technology effect}} - \underbrace{\frac{1}{\sigma}}_{\text{substitution effect}} \quad (76)$$

$$= \frac{\sigma-2-k}{\sigma k+1} \quad (77)$$

The overall effect of skill supply on wages is divided into two parts. The substitution effect comes from conventional labour demand theory. The directed technology effect, as labelled by Acemoglu (2002a), shows the effect of relative labour supply changes on wages through the adoption of skilled and unskilled technologies. The directed technology effect of skill supply on wages is unequivocally (weakly) positive. From corollary 10, $T$ falls after a rise in $Q$ if $\sigma < 1$. However, if $\sigma < 1$, a fall in $T$ would have a *positive* effect on the skill premium because of the rise in $p$ it would create (cf.(56)).

Also, if $\sigma > 1$, $T$ would rise and, although there would be a downward effect on $p$, it would not be big, so the overall effect is also *positive.* As discussed in section **??**, Acemoglu (2002b) distinguishes between skill *augmenting* technological change (a rise in $T$) and skill-*biased* technological change, which overall raises the marginal revenue product of skilled labour. Thus, a rise in $Q$ will lead to SBTC for any value of $\sigma \neq 1$.

Table 1: Summary of model predictions

| | SBTC | Rise in $W$ |
|---|---|---|
| Rise in $Q$ | $\sigma \neq 1$ | $\sigma > 2 + k$ |
| Rise in $R$ | $\sigma > 1$ | $\sigma > 1$ |

Table 2: International Elasticity of Substitution Estimates

| **Source** | **Region** | **Elasticity** |
|---|---|---|
| Berman & Machin (2000) | Large cross section of countries | 1.32 |
| Teal (2000) | Ghana | 0.28;2.2 |
| Fajnzylber & Maloney (2001) | Chile | 0.33 |
| Fajnzylber & Maloney (2001) | Mexico | 0.26 |
| Fajnzylber & Maloney (2001) | Colombia | 0.42 |
| Edwards & Behar (2006) | South Africa | 0.27 |

The overall effect of skill supply on wages is positive if $\sigma > 2 + \kappa$. Prior work for advanced countries by Acemoglu (2002a) produces thresholds for $\sigma$ that are up to and including 2. Our model produces a threshold that exceeds 2. This stems from differences in how we model the costs of importing/developing a new technology. In Acemoglu (ibid.), costs are constant or falling with every technology developed, depending on whether the specification conforms to the lab equipment or knowledge-based models of R&D in Rivera-Batiz & Romer (1991). The extent to which the threshold was below 2 depends on the degree of increasing returns to R&D, if any.

Our model has costs that increase in the acquisition of licences. As more licences are acquired, it becomes more expensive to purchase technologies closer to the frontier. We have argued why this is more appropriate than the established R&D models for developing countries because they do not engage in their own R&D. If we set $\kappa = 1$ such that the elasticity of cost with respect to the number of technologies imported is unity, the threshold is $\sigma = 3$. This suggests that, relative to a developed country, it is less likely that a developing country will experience a skill-induced rise in wage inequality.

**Proposition 14** *When skilled and unskilled labour are imperfect substitutes, NSBTC will lead to a rise in wage inequality iff* $\sigma > 1$.

**Proof.** Differentiating (57) with respect to $R$, using (67):

$$\frac{d \log W}{d \log R} = \frac{\partial \log W}{\partial \log T} \frac{d \log T}{d \log R} = \frac{\sigma - 1}{\sigma k + 1} \tag{78}$$

■

Recall NSBTC leads to an unequivocal rise in $T$. This translates to SBTC in the South, which has a net positive effect on the relative marginal revenue product of skilled labour, if $\sigma > 1$. We therefore see that the conditions required for global SBTC to generate a rise in the skill premium are less onerous than the conditions for a rise in skill supply to do so.

## 4.3 Model predictions

The values of $\sigma$ needed for rises in the skill composition or NSBTC in the North to cause SBTC and a rise in wage inequality in the South are summarised in table 1. We compare them with estimates of the elasticity of substitution, shown in table 2.

Using estimates of the technological parameters reported in Berman & Machin (2000), we calculate an elasticity of 1.32, as shown in the first row of table 2. Teal (2000) reports values of 0.28 (fixed effects) and 2.2 (no fixed effects). The South American labour demand studies did not have readily available measures of $\sigma$. However, we were able to calculate an elasticity of substitution measure using their estimates of labour demand elasticities.[8] Edwards & Behar (2006) use a CES technology to estimate an elasticity of 0.27, which is in line with other studies for particular economies in the table.

Comparing the elasticity estimates with the model predictions, we see the elasticities of less than unity for individual countries suggest NSBTC will not lead to a rise in wage inequality. The large international study by Berman & Machin (2000) suggests NSBTC could be an explanation for rising wage inequality in developing countries.

To examine the potential effects of skill supply on wage inequality, a natural threshold to consider has $\kappa = 1$ such that $\sigma > 3$. Acemoglu (1998) cites empirical examples of high elasticities of substitution (some exceeding his threshold of 2), but these do not appear to occur for a broad range of other developing countries. The values are *well* below the threshold of 3 for a rise in wage inequality to result.[9]. Therefore, even if we allow for (unequivocal) SBTC, a rise in skill supply will tend to reduce the skill premium and benefit the unskilled.

## 5 Concluding comments

Developing countries acquire licences for technologies that have been developed by technological leaders. This paper identifies two potential reasons why they may choose to develop technologies that favour skilled workers, which can lead to a rise in wage inequality. The first potential reason is that a rise in the supply of skilled workers makes it relatively more attractive to acquire the licence for a skilled technology rather than an unskilled one. If skilled and unskilled labour are perfect substitutes, this leads to a rise in the relative productivity of skilled workers and a rise in wage inequality. Unskilled workers lose out in absolute terms, not just relative to skilled workers. Modelling expanded educational access as a cohort-by-cohort rise in the skill supply can result in potentially long periods of stagnating unskilled wages.

If we relax the assumption that skilled and unskilled workers are perfect substitutes, a rise in skill supply still leads to SBTC. However, a high elasticity of substitution - one higher than a threshold that exceeds 2 - is required for a rise in wage inequality to result. Because it becomes more costly to acquire a new technology as a developing country gets closer to the frontier, this threshold is higher than for the advanced country model of Acemoglu (2002a). Therefore, a rise in wage inequality is less likely to result in a developing country after a rise in its supply of skills than in a developed country. Furthermore, comparisons with available empirical estimates suggest a rise in skill supply will not lead to a rise in wage in the skill premium.

The second potential reason for SBTC in the South is simply that SBTC takes place in the North. Consistent with the empirical observations in Berman & Machin (2000), the relative cost of acquiring a skilled technology is falling because the frontier of skilled technologies is advancing faster. If skilled and unskilled labour are perfect substitutes, the South experiences a rise in wage inequality because of SBTC in the North. If skilled and unskilled labour are imperfect substitutes, we find the conditions required for a rise in wage inequality to occur are not onerous and readily found in some of the empirical literature.

We conclude SBTC in the North is a more likely explanation for wage inequality than a domestic rise in skill supply is. However, two further remarks are in order. First, consistent with the directed technological change literature, a rise in skill supply in the North is a potential determinant of SBTC in the North. While a rise in skill supply may or may not lead to a rise in wage inequality in the North, it

[8] See Blackorby & Russell (1989).

[9] Only if we believe the estimates of Teal (2000) with no fixed effects and that $\kappa < 0.2$ can we find an example where a rise in skill supply will lead to a rise in wage inequality.

spurs SBTC in the North and potentially the South, resulting in a rise in wage inequality in the South. Second, many policies to expand education in the South are precisely a response to observed skill-biased shifts in labour demand. We proposed that a rise in skill supply, by stimulating further SBTC, might be self-defeating as far as reducing wage inequality is concerned. However, we find this is unlikely to be the case. Therefore, as far as wage inequality is concerned, the race between education and skill-biased technology is one that education can win.

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