Implications of Endogenous Group Formation for Efficient Risk-Sharing

Abstract

This paper models the implications of endogenous group formation for efficient risk-sharing contracts in the dynamic limited commitment model. Endogenising group formation requires that any risk-sharing arrangement is not only stable with respect to individual deviations but also with respect to deviations by sub-groups. This requirement alters the central predictions of the dynamic limited commitment model for efficient bilateral risk-sharing. Firstly, consumption of constrained agents depends on the previous history of shocks and the interaction of the history of shocks with the current income realizations of other constrained agents. As a consequence, the efficient contract does not display amnesia. Secondly, the covariance between current consumption and past income can take on negative values. Based on the first result, we derive a formal test for the presence of endogenous group formation under limited commitment. In addition, we show how this test can be extended to distinguish a limited commitment/perfect information environment from a full commitment/imperfect information environment empirically.

1 Introduction

As a response to the large fluctuations in their income, individuals in developing countries often enter into informal insurance arrangements that help them cope with risk. If these arrangements successfully replicate full insurance, then conditional on aggregate resources individual consumption allocations will not vary with own earnings. Barring other impediments, Pareto-efficient risk-sharing at the community level should be observed. Yet, empirical studies of risk-sharing uniformly reject the hypothesis of full insurance in rural communities (Deaton (1992), Townsend (1994), Udry (1994), Dercon and Krishnan (2000), Ligon et al. (2002), Fafchamps and Lund (2003), Murgai et al. (2002)). Instead they find that risk-sharing is limited to subgroups within the community and that even within such groups transfer behaviour departs from first best (Grimard (1997), Morduch (1991), De Weerdt and Dercon (2006)). The theoretical literature has proposed two explanations for such incomplete diversification: imperfect information and imperfect enforceability - usually with an emphasis on modelling risk-sharing as the sub-game perfect equilibrium of a 2-player game. Under imperfect information risk-sharing is limited by the fact that agents' income realizations are unobservable and the contract must be structured such that agents find it optimal to truthfully reveal their income realizations. Under imperfect enforceability, which is the focus of this paper, risk-sharing is limited because contracts are not legally enforceable ex-ante. Therefore, risk-sharing arrangements have to take into account that individuals will renege on the contract ex-post if the benefits from doing so outweigh the costs.

In the majority of the extant literature on imperfect enforceability, predictions on the pattern of individual consumption derived from bilateral models are translated to a multilateral environment maintaining the assumption that the contract must only be robust with respect to individual deviations. This may not be entirely satisfactory for two reasons: Firstly, contrary to what we observe, the individual deviation framework implies that any efficient risk-sharing arrangement in a homogenous population has to be at the level of the "community" or some other exogenously limited group (see Genicot and Ray (2003)). Secondly, one might suspect that predictions from a bilateral model do not necessarily extend to a multilateral environment when additional constraints arising from the fact that subgroups may jointly defect from the risk-sharing arrangement are modelled explicitly.

In this paper, we model an efficient dynamic risk-sharing contract in a multilateral environment that is required to be robust with respect to joint deviations by sub-groups. Indeed, we find that this requirement substantially alters the properties of the efficient risk-sharing contract. In our model all agents are infinitely lived and discount the future by a common factor. Income realizations in each period are i.i.d and publicly observed.[1] Insurance contracts cannot be legally enforced because courts cannot punish noncompliance. Such an environment may describe implicit insurance contracts within families, rural communities in developing countries as well as a model of international sovereign debt. Since agents are risk-averse, they will always be willing to share risk ex-ante. However, since they cannot precommit, any agent with a high endowment realization has an incentive to renege when called upon to help others. To define the coalition-proof equilibrium contract, we must specify what happens out of equilibrium. We assume that any group of agents who renege on an insurance contract are observed by all others. Furthermore, they are thereafter forever excluded from the existing arrangement, but the reneging group may continue risk-sharing among themselves after breaching the contract provided they are themselves immune to further deviations. This seems a plausible assumption. After all, if a large group can reach an agreement to share risk, a smaller group can do so too and threaten to deviate jointly instead of continuing in autarky after deviation.

The theory developed here is related to several other papers in the literature. Ligon et al. (2002) and Kocherlakota (1996) have characterized the efficient sub-game perfect risk-sharing contract in a two-player game. Attanasio and Rios-Rull (2000) have extended this model to an environment

[1] As pointed out by Udry (1994), imperfect enforceability rather than limited information appears to be the binding constraint in village economies and we therefore assume perfect information.

with limited commitment and public insurance. The above authors find that the optimal contract is non-stationary and exhibits positive history dependence. An agent with a high income realization is induced to share his good fortune in the current period by being offered future rewards in the form of at least partial repayment (Fafchamps (1999)). Ligon et al. (2002) demonstrate that the sub-game perfect contract displays history-dependence only when enforcement constraints are slack. When enforcement constraints are binding, the contract is said to display amnesia: the previous history of income realizations becomes irrelevant and consumption and future payoffs depend only on the current income realization because allocations are determined by the players' autarky payoffs. To define the set of coalition-proof contracts we adopt the stability concept derived in Genicot and Ray (2003), who characterize the optimal stationary risk-sharing contract in an insurance group which is robust with respect to deviations by individuals as well as subgroups. This requirement endogenizes the size of insurance groups and the authors show that the size of stable insurance groups in the class of all history dependent and possibly asymmetric contracts is bounded.

In this paper, we solve for the efficient contract in the set of all coalition-proof history-dependent contracts and describe its analytical and statistical properties. We find that many of the attributes that have come to be associated with efficient risk-sharing are in fact a consequence of modelling risk-sharing as the sub-game perfect equilibrium of a repeated game. That is to say, they are not robust when equilibria are refined to be coalition-proof – a subset of the set of all sub-game perfect equilibria. Most importantly, we prove that an efficient coalition-proof contract is history-dependent both when enforcement constraints are slack and when they are binding. This is the case, because consumption and continuation payoffs have to be adjusted so that agents are indifferent between deviation followed by reversion to the punishment payoff and remaining in the risk-sharing arrangement when enforcement constraints are binding. In a sub-game perfect contract, the punishment path entails reversion to the static autarky equilibrium. As a consequence, the previous history of shocks, which is summarized by the ratios of marginal utilities between agents in the previous period, becomes irrelevant when enforcement constraints are binding. However, the autarky payoff is not renegotiation-proof in a multi-player environment, since any subgroup of excluded players can do better by abandoning the autarky equilibrium and instead continue risk-sharing. Instead, we prove that it is optimal to satisfy the enforcement constraints for a single point on the Pareto frontier of the set of coalition-proof contracts for any stable subgroup. We show that this condition involves trading off the punishment paths of all constrained agents according to their previous history of shocks in a way that involves the least change in the marginal utility ratios. This implies that consumption of constrained agents depends on the previous history of shocks and the interaction of the history of shocks with the current income realizations of other constrained agents.

There are two empirical implications of endogenizing the group formation process through the requirement of coalition-proofness: Firstly, the covariance between current consumption and past income conditional on past aggregate income is no longer constrained to be nonnegative in a coalition-proof contract. This is the case because an increase in a constrained agent's income realization may raise the consumption of other constrained agents (and therefore reduce own consumption) by increasing the set of joint deviations that are profitable. If this effect outweighs the positive effect of own income on own consumption, then current consumption may decrease with changes in past income realizations and this property of a negative history dependence provides a way to distinguish between a coalition-proof and a sub-game perfect contract under limited commitment.

Secondly, the fact that the coalition-proof contract no longer displays amnesia can be used to test for endogenous group formation. The test is based on the observation that under endogenous group formation, the current consumption of a constrained household depends on the previous marginal utility ratios of the entire set of constrained agents both directly and interacted with their current income realization. Under standard assumptions on the utility function, this can be tested using a panel of consumption and income data on all members of an informal insurance arrangement. Dubois (2005) derives a test for the likelihood of endogenous insurance group sizes with heterogenous preferences under the assumption of stationary symmetric risk-sharing. Chaudhuri et al. (2005) and Ahn et al. (2006) present tests in an experimental setting. In contrast, the test presented here is valid in a non-experimental dynamic setting. It relies on being able to identify the Pareto or bargaining weights of individuals in a risk-sharing arrangement in each period and is thus similar in spirit to Mazzocco (2007) who derives a test for limited versus full commitment in a bilateral setting.

Finally, we also demonstrate how to determine whether imperfect commitment or imperfect observability impede first-best risk-sharing. In general, distinguishing the two environments empirically is a difficult task. Indeed, Wang (1995) shows that a contract that is constrained by asymmetric information shares many of the same features as a contract constrained by limited commitment. However, the two types of contracts display some subtle differences which can help to distinguish them empirically. In the limited commitment contract, choosing consumption in the case of binding constraints involves trading off the consumption of a subcoalition of constrained agents. In the limited information contract, choosing consumption involves trading off a constrained agent's consumption across different states. As we show, this property can be used to distinguish empirically between the two contracts.

The paper proceeds as follows: Section 2 outlines the model and its assumptions. Section 3 presents the analytical results. Section 4 derives empirical results and presents a test for the endogeneity of group formation. Section 5 implements the test using computed model solutions. Section 6 concludes.

2 The Model

There is a set of $\mathbb{N} = \{1, ..., n\}$ households in a community. Each period $t = 1, 2, ...\infty$, household i receives an income $y^i(s_t)$, where $s = 1, ..., S$ is the independently and identically distributed state of nature, which occurs with probability π_s. All households have an identical twice continuously differentiable utility function $u(c^i(s_t))$ where $c^i(s_t)$ is consumption. Households are risk-averse, infinitely lived and discount the future with common discount factor β. Perfect information is assumed and there are no opportunities for storage. Risk-sharing contracts must be self-enforcing, which requires that at any point in time the long-run benefit from complying with the contract must outweigh the short-run gain from reneging.

To define the set of self-enforcing or stable contracts, we follow Genicot and Ray (2003) and assess stability of a risk-sharing group of size n recursively by first examining the stability of groups of size $1, ..., n-1$ and then checking whether a risk-sharing contract exists for a group of size n that is robust with respect to deviations by stable subgroups. Noncompliance with a risk-sharing arrangement has the following effects.

Assumption 1 *Any sub-group of agents who renege consume their autarky income during the period of deviation. The sub-group is forever excluded from the existing arrangement, but may continue risk-sharing amongst themselves after breaching the contract.*[2]

Without savings a single household will consume its income in each period. The expected value of autarky is

$$V^*(1) = E\sum_{t=0}^{\infty}\beta^t u(y_t). \tag{1}$$

[2]It is also possible to assume that agents who deviate continue insurance in the deviating subgroup during the period of deviation. None of the results in this paper are affected by this.

This is the only stable payoff for an individual and hence the set of stable payoffs contains just one element $\mathbb{V}^*(1) = \{V^*(1)\}$.

Having found the autarky payoff, we proceed to risk-sharing in groups. Supposing we have defined stable sets of expected payoffs $\mathbb{V}^*(m)$ for all $m = 1, ..., n-1$, where each element in $\mathbb{V}^*(m)$ is a vector of size m with components $V = \{V^1, V^2, ..., V^m\}$, we can now assess the stability of a risk-sharing arrangement of size n. An insurance contract ω is a mapping from histories of exogenous shocks to transfers. Let s_t be the state of the world occurring at date t. The contract specifies for every date t and every history of states s, $h_t = (s_1, s_2, ..., s_t)$, a consumption allocation $c^i(h_t)$ for each household $i = 1, ..., n$. For any history h_t and contract ω and group size m, the lifetime utility of an individual from time t onwards is

$$U^i(\omega, h_t, m) = u(c^i_t)) + E \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} u(c^i_\tau). \tag{2}$$

A contract for a group of size n is sub-game perfect if no unilateral deviations are profitable, which corresponds to playing the worst possible equilibrium during the punishment phase (Abreu (1988)). However, such a contract is not coalition-proof, since any subcoalition of players can achieve a Pareto improvement by abandoning the original punishment of autarky and instead continuing risk-sharing (see Bernheim et al. (1987)). Therefore an insurance scheme is defined as stable if no history of states exists, for which a stable subgroup could credibly deviate from the arrangement, consume autarky income in the period of deviation and then continue insurance within the subgroup. Credibility means that a deviating subgroup must itself be immune to further deviations (see Bernheim et al. (1987) and Bernheim and Ray (1989)). Stability of a risk-sharing arrangement of size n is then expressed in the following two conditions stated in Genicot and Ray (2003):

Definition 1 *V is a stable payoff vector for n, if the following two conditions are met:*

[PARTICIPATION] For no history h_t is there a sub-group of individuals $m < n$ and a stable payoff vector $V \in \mathbb{V}^*(m)$ such that $V^i(n) < V^i(m)$ for all $i = 1, ..., m$.

[ENFORCEMENT] For no history h_t is there a subset $\mathbb{M}$ of individuals of size $m < n$ and a stable payoff vector $V \in \mathbb{V}^*(m)$ such that for all $i \in m$

$$u(y^i(s_t)) + \beta V^i > u(c^i(s_t)) + \beta V^i(h_{t+1}, n), \tag{3}$$

where h_{t+1} is the history h_t concatenated with next period's income realizations.

If the payoff vector V is stable, then the risk-sharing contract that generates this payoff vector for the group of size n is self-enforcing. In particular, a stable $V(n)$ constitutes a coalition-proof equilibrium of the risk-sharing game.

Definition 2 *Let $\Omega(h_t)$ be the set of self-enforcing contracts after history h_t that satisfy the enforcement and the participation constraint.*

The enforcement constraints in (3) are defined negatively. To make this definition of stability operational as part of a maximization problem, the enforcement constraints must be written in a more succinct manner. Suppose, we have found the sets of stable contracts for all $m < n$, and in addition have solved for the Pareto frontier of these sets. We now argue that it is not necessary to satisfy (3) for all $V \in \mathbb{V}^*(m)$. Instead, it is sufficient to satisfy the enforcement constraints for a single point on the Pareto frontier of $\mathbb{V}^*(m)$ augmented by one period of autarky consumption. To see this, consider a risk-sharing group of size n and suppose that there is an efficient allocation $\widetilde{V}$ on the Pareto frontier of $\mathbb{V}^*(2)$ such that agent 1 and 2 find it profitable to deviate. To deter this deviation, consumption and expected continuation payoffs for agent 1 and 2 in the existing arrangement must be set such that

$$u(y_s^1) + \beta\widetilde{V}^1 = u(c_s^1) + \beta V^1(h_{t+1}, n) \tag{4}$$

$$u(y_s^2) + \beta\widetilde{V}^2 = u(c_s^2) + \beta V^2(h_{t+1}, n). \tag{5}$$

After consumption and continuation payoffs have been updated to satisfy the enforcement constraints at equality for $\widetilde{V}$, there can be no joint deviation by agent 1 and 2 to a different V' on the Pareto frontier of $\mathbb{V}^*(2)$ that would render n unstable. By definition, any move along the Pareto frontier makes at least one agent worse off and for this agent a deviation is no longer profitable. Since the enforcement constraints state that stability of n is only threatened if all members of a deviating subgroup are better off following a joint deviation, threatening to renege and continue with $V' \neq \widetilde{V}$ is not credible once consumption and continuation payoffs have been adjusted to $\widetilde{V}$. An analogous argument can be made for any subgroup $\mathbb{M} \in \mathbb{N}$.[3]

[3] Of course one may argue that the deviating group could use side-payments to trade to the symmetric allocation V', which gives the largest total sum of utility. However, in this case the arrangement of size n should be able to use side-payments as well and this would alleviate the self-enforcing constraints for all group sizes. Since we want to focus on the effect of imperfect enforceability, we rule out side payments.

3 Efficient Contracts

Definition 3 *A contract $\omega(h_t) \in \Omega(h_t)$ is efficient if there exists no other self-enforcing contract that gives all agents at least as much utility and one agent strictly more.*

We follow Ligon et al. (2002) in writing the maximization problem recursively in terms of ex-post promised values that are conditional on the current period income realization and summarize the relevant information about each player's history (see Abreu et al. (1990) and Phelan and Townsend (1991)). We define $U_s^n(U_s^1, U_s^2, ..., U_s^{n-1})$ to be the ex-post Pareto frontier which solves the problem of maximizing agent n's lifetime discounted utility subject to agent $i \neq n$ receiving at least discounted utility U_s^i. This problem can be thought of as a social planner having promised agent $1, ..., n-1$ utility $U_s^1, ..., U_s^{n-1}$, which the planner delivers by choosing current consumption and continuation utilities in a way that maximises agent n's payoff and satisfies the self-enforcing constraints. In addition, the planner can choose a punishment path $\widetilde{V}$ subject to $\widetilde{V}$ being compatible with the payoff that can be achieved by deviating implicit in the equilibrium concept employed: in a coalition-proof equilibrium $\widetilde{V}$ must correspond to a point on the Pareto frontier of any stable subgroup $\mathbb{M} \in \mathbb{N}$ that threatens to deviate.

This discussion gives rise to the following dynamic programme

$$U_s^n(U_s^1, U_s^2, ..., U_s^{n-1}) = \max_{((U_r^i)_{r=1}^S)_{i=1}^{n-1}, (c_s^i)_{i=1}^n, \widetilde{V}} u(c_s^n) + \beta \sum_{r=1}^{S} \pi_r U_r^n \left(U_r^1, ..., U_r^{n-1}\right) \tag{6}$$

subject to a set of promise-keeping constraints

$$\gamma^i \quad : \quad u(c_s^i) + \beta \sum_{r=1}^{S} \pi_r U_r^i \geq U_s^i \quad \forall i \neq n \tag{7}$$

and an aggregate resource constraint

$$\eta \quad : \quad \sum_{i=1}^{n} y_s^i \geq \sum_{i=1}^{n} c_s^i.$$

The solution must also be sustainable - i.e the group of size n must be stable - and so satisfy the enforcement constraints for any stable group of size $m < n$:

$$\beta\gamma^i \pi_{sr} \phi_r^i \quad : \quad U_r^i \geq u(y_r^i) + \beta \widetilde{V}^i \quad \forall \mathbb{M} \subset \mathbb{N} \quad s.t. |\mathbb{M}| = m, \quad \forall r \in S \tag{8}$$

$$\beta \pi_{sr} \phi_r^n \quad : \quad U_r^n(U_r^1, U_r^2, ..., U_r^{n-1}) \geq u(y_r^n) + \beta \widetilde{V}^n \quad \forall \mathbb{M} \subset \mathbb{N} \quad s.t. |\mathbb{M}| = m, \quad \forall r \in S. \tag{9}$$

In order to characterize the solution to this dynamic programming problem further, we need to establish the shape of the Pareto frontier and its domain of definition. For the multilateral subgame perfect contract which is stable with respect to individual deviations, the results established in Lemma 1 in Thomas and Worrall (1988) apply mutatis mutandis. Since the set of sustainable contracts is not convex in the coalition-proof contract we cannot rely on this property to establish the concavity and differentiability of the value function. Instead, we can apply Corollary 4 and 5 in Milgrom and Segal (2002) and Theorem 2 in Cotter and Park (2006) to establish differentiability of the value function. These technical results are summarized in the following Lemma:

Lemma 1

1. *The Pareto frontier has a right and left derivative everywhere and its directional derivatives equal:*

$$\begin{aligned} \frac{\partial U_s^{n,+}}{\partial U_s^i} &= -\min \gamma_s^i \\ \frac{\partial U_s^{n,-}}{\partial U_s^i} &= -\max \gamma_s^i \end{aligned} \tag{10}$$

2. *The Pareto frontier $U_s^n(U_s^1, ..., U_s^{n-1})$ is differentiable for any $U_s^1, ..., U_s^{n-1}$ on the range of the optimal policy correspondence.*

3. *If $U_r^{i'} > U_r^i$, then $\frac{\partial U_r^n}{\partial U_r^i} > \frac{\partial U_r^n}{\partial U_r^{i'}}$ on the range of the optimal policy correspondence.*

Proof of Lemma 1

See Appendix.

The first-order conditions of the dynamic programming problem are

$$\frac{u'(c_s^n)}{u'(c_s^i)} = \gamma^i \quad \forall i \neq n \tag{11}$$

and

$$-\frac{\partial U_r^n}{\partial U_r^i} = \gamma^i \frac{1 + \phi_r^i}{1 + \phi_r^n} \quad \forall r \in S, \quad \forall i \neq n \tag{12}$$

together with the $n - 1$ envelope conditions

$$\gamma^i = -\frac{\partial U_s^n}{\partial U_s^i} \quad \forall i \neq n. \tag{13}$$

Advancing the envelope condition forward by one period, we have

$$\gamma_r^i = -\frac{\partial U_r^n}{\partial U_r^i} = \gamma^i \frac{1+\phi_r^i}{1+\phi_r^n} = \frac{u'(c_r^n)}{u'(c_r^i)} \quad \forall r \in S, \quad \forall i \neq n. \tag{14}$$

That is, γ^i measures the trade-off between agent n's and agent i's discounted lifetime utility in the current period, and γ_r^i measures the trade-off in the future period when the state of the world is r.

More generally, the first order condition shows that a constrained-efficient contract can be characterized in terms of the evolution of the vector $\gamma = \{\gamma^1, \gamma^2, ..., \gamma^{n-1}\}$. From the first order condition, each γ^i measures the ratio of marginal utilities between agent i and n and from the envelope condition, each γ^i measures the rate at which the discounted utility of agent i can be traded off against the discounted utility of agent n keeping everybody else's utility constant. Once the state of nature r is known, the new value for each γ_r^i is determined from equation (12) and (14). The first-order conditions are interpreted as follows: First suppose that there are no binding enforcement constraints, so all ϕ_r^i are zero. In this case, (12) prescribes that γ_r^i remains constant and consequently, so does the ratio of marginal utilities and the rate at which continuation utilities are traded off. Now suppose agent i has a binding constraint in state r and agent n is unconstrained, then equation (14) states that $\gamma_r^i > \gamma^i$ and consequently $\frac{u'(c_r^n)}{u'(c_r^i)} > \frac{u'(c_s^n)}{u'(c_s^i)}$. By the concavity of the utility function, this implies that consumption growth for agent i is greater than for agent n. Equally, $\frac{\partial U_r^n}{\partial U_r^i} < \frac{\partial U_s^n}{\partial U_s^i}$ and from Lemma 1, this implies that agent i's continuation payoff is increased at the expense of agent n in the face of a binding constraint.

Thus, the constrained efficient contract introduces an element of 'quasi-credit' (see Fafchamps (1999)). An agent most likely faces a binding constraint when he has experienced a high income realization and is called upon to make a positive transfer to someone less fortunate. The optimal contract induces risk-sharing by offering the agent a future reward in the form of a higher continuation payoff and this increase is maintained as long as no other agent experiences a binding constraint. In other words, the agent is encouraged to transfer more at present through the promise that some of this transfer will be repaid in the future.

The main difference between a sub-game perfect contract and a coalition-proof contract arises when enforcement constraints are binding.[4]

[4] In the following sub-game perfect refers to a contract that is sub-game perfect but not necessarily coalition-proof, while coalition-proof refers to a contract that is both sub-game perfect and coalition-proof.

Proposition 1

1. *In a sub-game perfect efficient contract, the planner will choose the punishment path* $\widetilde{V}^i = V^*(1)$ *for all* $i = 1, ..., n$*. As a result, payoffs in this contract are not history-dependent for agents with binding enforcement constraints.*

2. *In a coalition-proof efficient contract, the planner will choose a punishment path* $\widetilde{V}$ *on the Pareto frontier of* $V^*(m)$ *for any stable group of size* m*. The choice of* $\widetilde{V}$ *depends on the history of income realizations. Therefore, payoffs* U_r^i *of constrained agents in the coalition-proof efficient contract are history-dependent for agents with binding enforcement constraints.*

3. *In a sub-game perfect efficient contract, the payoff* U_r^i *of a constrained agent depends only on his own current income realization.*

4. *In a coalition-proof efficient contract, the payoff* U_r^i *of a constrained agent depends on his own current income realization as well as that of other constrained agents. The effect of* y_r^j *on* U_r^i *is greater the smaller* $|\gamma^i - \gamma^j|$*.*

Proof of Proposition 1: See Appendix.

The proof proceeds by recasting the dynamic programme in (6) as a social planner problem of maximizing a weighted sum of utilities with planning weights $\sum_{i=1}^{n} \lambda^i = 1$ subject to the usual enforcement constraints. The planning weights in this contract are related to the Lagrange multipliers on the promise keeping constraints by $\gamma^i = \frac{\lambda^i}{\lambda^n}$. In the sub-game perfect contract, the planner's value function is maximized by satisfying the enforcement constraints for the autarky payoff for all constrained agents since this involves the smallest movement in the ratio of marginal utilities. Since the autarky payoff depends only on the agent's own current income realization, the contract displays amnesia: The previous history of shocks as well as the income realizations of other constrained agents are irrelevant in determining U_r^i for a constrained agent. In the coalition-proof contract, the choice of punishment paths and therefore the continuation payoffs involves a trade-off between constrained agents. Suppose agent 1 and 2 are constrained and the previous history of shocks has resulted in initial planning weights such that $\lambda^2 > \lambda^1$ and $y_r^1 = y_r^2$. Then it is optimal to deter a joint deviation by choosing a punishment path on the Pareto frontier of a stable group of size 2 that gives a higher continuation value to agent 2. This is how the history of income realizations up to time $t-1$ affects the efficient coalition-proof contract when enforcement constraints are binding.

To prove claim (4) consider the scenario in which both agent 1 and 2 are constrained, $\lambda^2 > \lambda^1$ and both agents have a low income realization $y_r^1 = y_r^2 = y_l$. Then it follows from the argument made in part (2) of the proposition that $U_r^1 = u(y_l) + \beta \widetilde{V}^1 < U_r^2 = u(y_l) + \beta \widetilde{V}^2$. If agent 2's income now increases to $y_r^2 = y_h$, it can be shown that there is an enforceable contract which is a convex combination of the original choice $U_r^1 = u(y_l) + \beta \widetilde{V}^1$ and $U_r^2 = u(y_h) + \beta \widetilde{V}^2$. By the concavity of the Pareto frontier, this contract increases total utility if λ^1 and λ^2 are of similar size and hence the allocation of $U_r^1 = u(y_l) + \beta \widetilde{V}^1$ for agent 1 can no longer be optimal. Intuitively, when initial Pareto weights of two constrained agents are similar, it is optimal for the planner to align their consumption by choosing a point on the Pareto frontier for a group of 2 in which the agent with the lower current period income realization receives a larger share of resources from tomorrow onwards.

It is now easy to see intuitively why the optimal continuation payoffs are history-dependent in the face of binding constraints as shown in Proposition 1. An agent i arrives with a high promised value and a correspondingly high Pareto weight and consumption share in period t only, if he has been a net giver in the risk-sharing arrangement over time. Conversely, an agent k with a low Pareto weight has been a net receiver over time and agent i has shared income in earlier periods precisely because of the promise of future repayments in period t. Of course, this tradeoff between current consumption and future utility is feasible only to the extent that agent k finds it in his interest to repay the loan, i.e. his continuation payoff must not be so low in any state in the future that it is optimal to renege. Hence, following any histories in which i has been a net giver to k, it is optimal to deter joint deviations by i and k by satisfying the enforcement constraints for punishment paths which give i a relatively higher payoff than k. The opposite is true for histories in which k has been a net giver. Choosing future payoffs in this way, which minimizes the movement in the ratio of marginal utilities, increases the extent of insurance in the current period because it increases the scope for trading off current consumption against future utility and therefore has a positive effect on overall welfare.

Combining Proposition 1 and the first order conditions (11)-(14), we can see how consumption of a constrained agent is linked to the continuation payoffs and income realizations of other constrained agents. Denote by C the set of constrained agents and UC the set of unconstrained agents. Define $\underline{U}_r^i$ as the payoff of agent i, when his enforcement constraint is binding. In a sub-game perfect contract, this payoff depends only on his own income realization. In the coalition-proof contract, it depends on the income realization of other constrained agents as well as the previous history of the contract. This implies that consumption c^i of a constrained agent is determined by

$$\frac{u'(c_r^n)}{u'(c_r^i)} = -\frac{\partial U_r^n}{\partial U_r^i}\Big|_{U_r^i=\underline{U}_r^i(y_r^i),U_r^{j\in C}=\underline{U}_r^j(y_r^j),U_r^{k\in UC}=U_r^k(h_t)}$$

in the sub-game perfect contract. In contrast, in the case of the coalition-proof contract, consumption of a constrained agent c^i is given by

$$\frac{u'(c_r^n)}{u'(c_r^i)} = -\frac{\partial U_r^n}{\partial U_r^i}\Big|_{U_r^i=\underline{U}_r^i(y_r^i,\{y_r^j\}_{j\in C},h_t),U_r^{j\in C}=\underline{U}_r^j(y_r^i,\{y_r^j\}_{j\in C},h_t),U_r^{k\in UC}=U_r^k(h_t)}.$$

As a consequence, consumption depends on the previous history of constrained agents only in the coalition-proof contract. The income realization of other constrained agents enters both in the sub-game perfect and in the coalition-proof contract. However, in the former it enters only indirectly via its effect on U_r^j whereas in the latter, it directly affects U_r^i and the magnitude of this effect depends on the previous history of shocks as outlined in Proposition 1.

4 Statistical Properties

In this section, we will demonstrate that in a coalition-proof efficient contract the covariance between consumption and past income conditional on aggregate resources can take on negative values. This property is unique to the coalition-proof limited commitment contract and hence can be used to distinguish between the sub-game perfect and the coalition-proof contract, which correspond to exogenous and endogenous group sizes respectively. Secondly, we exploit the property that history enters both directly and interacted with current income realizations into the first order condition for consumption of a constrained agent in the coalition-proof contract to derive a test for the endogeneity of insurance group size in an environment of perfect information. Finally, we discuss, how these empirical properties can be used to distinguish an environment of limited commitment from an environment of limited information.

The sign of the conditional covariance is characterized in Proposition 2.

Proposition 2

1. $\text{Cov}(c_t^i, y_{t-1}^i | (y_t^j)_{j=1}^{n-1}, Y_t, (y_{t-1}^j)_{j\neq i}^{n-1}, Y_{t-1}, (U_{t-2}^j)_{j=1}^{n-1}) \geq 0$ *in a sub-game perfect efficient risk-sharing contract.*

2. $\text{Cov}(c_t^i, y_{t-1}^i | (y_t^j)_{j=1}^{n-1}, Y_t, (y_{t-1}^j)_{j\neq i}^{n-1}, Y_{t-1}, (U_{t-2}^j)_{j=1}^{n-1}) \gtreqless 0$ *in a coalition-proof efficient risk-sharing contract.*

Proof of Proposition 2: See Appendix.

To see this, note that from the first order conditions (11)–(14), the effect of a change in y_{t-1}^i on c_t^i is given by

$$\frac{dc_t^i}{dy_{t-1}^i} = \sum_{j=1}^{n-1} \frac{\partial c_t^i}{\partial U_{t-1}^j} \frac{\partial U_{t-1}^j}{\partial y_{t-1}^i}. \tag{15}$$

In the sub-game perfect contract, all the above terms are nonnegative. However, this is not necessarily the case in the coalition-proof contract. To see this, consider the following scenario: Suppose there are two income states and agent 2 finds it profitable to deviate either individually or jointly with agent 1 in both states in period $t-1$. If agent 1 has a low income realization, he does not find a deviation profitable, but moving to the high income state he does. This means the planner has to deter an individual deviation of agent 2 when agent 1 has low income and a joint deviation when agent 1 has high income, which means that in such a configuration agent 2 is better off under the high income realization for agent 1. Therefore $\frac{\partial U_{t-1}^j}{\partial y_{t-1}^i} > 0$. Now suppose, both agents have a binding constraint in period t. Since agent 2's Pareto weight in $t-1$ is higher following the high income realization of agent 1, Proposition 1 tells us that the planner will choose to satisfy the enforcement constraints for a point on the Pareto frontier of a stable subgroup of size 2 that gives a relatively larger allocation to agent 2. If this effect is large enough, then agent 1's consumption in period t following the high income realization in period $t-1$ may be lower than following the low income realization. Such a situation cannot arise in the sub-game perfect contract because an agent's deviation payoff does not depend on the income realization of other agents.

Repeatedly applying the law of iterated expectations as in Proposition 3.1 in Kocherlakota (1996), it then follows that the sign of $\text{Cov}(c_t^i, y_{t-1}^i | \sum_{j\neq i}^n y_{t-1}^j, Y_{t-1})$ is determined by the sign of

$$\text{Cov}(c_t^i, y_{t-1}^i | (y_t^j)_{j=1}^{n-1}, Y_t, (y_{t-1}^j)_{j\neq i}^{n-1}, Y_{t-1}, (U_{t-2}^j)_{j=1}^{n-1}).$$

Detecting a significant negative covariance between current consumption and past income would therefore indicate that the group formation process is endogenous.

Secondly, it is shown how to test for the endogeneity of group formation in an environment of limited commitment and perfect information given T observations over time on the consumption $(c_t^i)_{i=1}^n$ and income $(y_t^i)_{i=1}^n$ of n people in a risk-sharing pool. Kocherlakota (1996) has shown that it is possible to sort the households into a set of constrained households C_t and a set of unconstrained households UC_t in each period t by using information on the evolution of their marginal utility ratios over time. The sorting procedure is valid under quite general assumptions about the form of the utility function. Quite generally, it follows from the first order conditions that

$$(c_t^i) = g(\gamma_{t-1}, y_t). \tag{16}$$

where $\gamma = \{\gamma^1, ..., \gamma^{n-1}\}$ is the $n-1$ dimensional vector, whose i'th element is the marginal utility ratio between household i and household n and $y = \{y^1, ..., y^n\}$ is the vector of income realizations.

If coalitional deviations do not present a threat to the stability of insurance arrangements, then current consumption of a constrained household i is a function of current income of constrained agents and the marginal utility ratios γ^i of only those agents that are unconstrained in period t:

$$(c_t^i)_{i \in C_t} = g((\gamma_{t-1}^m)_{m \in UC_t}, (y_t^k)_{k \neq i \in C_t}, y_t^i, Y_t) \tag{17}$$

where Y_t is aggregate income.

In the coalition-proof contract, the marginal utility ratio of constrained agents as well as their current incomes determine the joint payoff in case of deviation. The implication is that the consumption of agents, who are constrained, will depend on the marginal utility ratios and incomes of all agents $k \in C_t$ and in particular that the effect of income is larger if the marginal utility ratios of two constrained agents are of similar magnitude:

$$(c_t^i)_{i \in C_t} = g((\gamma_{t-1}^m)_{m \in UC_t}, (\gamma_{t-1}^k)_{k \in C_t}, (y_t^j, |\gamma_{t-1}^i - \gamma_{t-1}^j| \times y_t^j)_{j \neq i \in C_t}, y_t^i, , Y_t). \tag{18}$$

This prediction can be tested by estimating the above as a linear regression and testing for significance of $(\gamma_{t-1}^k)_{k \in C_t}$ and $(|\gamma_{t-1}^i - \gamma_{t-1}^j| \times y_t^j)_{j \neq i \in C_t}$.

Of course, we may not only want to distinguish whether insurance is limited by individual deviations or by group deviations but also whether limited commitment or limited information drives the departure from first best. In general, distinguishing the two environments empirically is a difficult task. Indeed, Wang (1995) shows that the requirement to elicit truth-telling from agents in a bilateral constrained efficient contract under imperfect information results in individual consumption being positively correlated with current individual income and being potentially related to lagged realizations of both individuals' incomes. Thus, the implications of efficiency in full information/no-commitment environments are qualitatively similar to the implications of efficiency in asymmetric-information/full-commitment environments.

However, the two types of contracts display some subtle differences which can help to distinguish them empirically. Under limited commitment, the prescription in case of a binding constraint is clear: In the case of individual deviations, the contract displays amnesia. In the case of group deviations, satisfying the enforcement constraints involves trading off the utilities of constrained agents against each other taking account of the past history as well as the income realizations of other constrained agents. In the case of an information constrained contract, the prescription is quite different when constraints are binding. Firstly, both the left-hand side as well as the right-hand side of the enforcement constraint contain the expected life-time utility of being in a group of size n. Therefore the optimal contract will not display amnesia because payoffs both on and off the equilibrium path are dynamic. Secondly, because the deviation payoff of agent i depends on the entire group n and not on the subcoalition of constrained agents, it should not be directly affected by the other constrained agents. Thirdly, raising consumption in state s_1 and thereby increasing the utility from risk-sharing in s_1 makes it more profitable to pretend in state s_2 that state s_1 has in fact occurred. To put it another way, increasing consumption in state s_1 increases the deviation payoff in state s_2. Hence, choosing consumption in the case of binding constraints involves trading off a constrained agent's consumption across different states *not* trading off the consumption of several constrained agents for a given state. Proving these results analytically for the asymmetric information contract goes beyond the scope of this paper, however, we now present numerical results which suggest that it is possible to distinguish the different environments empirically.

5 Simulations and Testing

In this section, we use simulated model solutions to show how to discriminate between endogenous and exogenous group formation under imperfect enforceability and how to distinguish whether first best risk-sharing is impeded by limited commitment or limited information.

To solve the model numerically, we restrict ourselves to 3 households with logarithmic utility functions. Individual income y^i takes on the values 2,4 and 6, with equal probability. Income realisations across households are identically and independently distributed. The discount factor is $\beta = 0.85$. To compute the optimal contract, we follow Marcet and Marimon (1999) and Attanasio and Rios-Rull (2000) in setting up a Lagrangian for a social planner who seeks to maximize a weighted sum of utilities of a group of n agents by choice of current consumption and future Pareto weights. We implement the algorithm using weighted-residual methods based on finite element approximations to solve for the equilibrium of this model in each income state on a two dimensional grid of Pareto weights, which are normalized such that $\sum_{i=1}^{3} \lambda^i = 1$ (see Judd (1998), Judd (1992), McGrattan (1996)). We solve for both the efficient sub-game perfect contract, which corresponds to exogenous group formation, as well as for the efficient coalition-proof contract, which corresponds to endogenous group formation.

To recap, the results in Proposition 1 show that:

1. The past history of shocks of constrained agents contains information about the current consumption of constrained agents in the coalition-proof efficient contract.

2. Current consumption of constrained agents corresponds more strongly to changes in the income realization of other agents in the coalition-proof contract and the strength of the response depends on the initial marginal utility ratio between constrained agents.

It is precisely these two properties of the coalition-proof efficient contract that provide a test for the presence of endogenous group formation in a symmetric perfect information/ no-commitment environment as shown in Section 4.

As our test only applies to constrained agents, we construct a vector consisting of agent 1's consumption evaluated at 1000 separate grid points where both agent 1 and agent 2 have a binding constraint and add a random error $\epsilon \sim N(0, 0.01)$ to each element of the vector. We then run the following regressions:

$$c_t^1 = \beta_0 + \beta_1 \lambda_{t-1}^1 + \beta_2 \lambda_{t-1}^2 + \beta_3 y_t^1 + \beta_4 y_t^2 + \beta_5 Y_t \quad (19)$$

$$c_t^1 = \beta_0 + \beta_1 \lambda_{t-1}^1 + \beta_2 \lambda_{t-1}^2 + \beta_3 y_t^1 + \beta_4 y_t^2 + \beta_5 Y_t + \beta_6 |\lambda_{t-1}^1 - \lambda_{t-1}^2| \times y_t^2 \quad (20)$$

where λ_{t-1}^1 and λ_{t-1}^2 are the initial Pareto weights of agent 1 and 2, which are related to the marginal utility ratios in (18) via $\lambda^i / \lambda^n = \gamma^i$ and are equal to the consumption shares for a logarithmic utility function. In (19) we test whether the past history of constrained agents as summarized by λ_{t-1}^1 and λ_{t-1}^2 is significant in explaining own consumption. If group formation is exogenous, the coefficient on λ^1_{t-1} and on λ^2_{t-1} is expected to be zero. If group formation is endogenous, then as λ^2_{t-1} increases relative to λ^1_{t-1}, we would expect agent 2 to be awarded a higher payoff than agent 1 when enforcement constraints are binding, because this requires a smaller movement in the marginal utility ratios. Hence the two coefficients ought to be significant and have opposite effects. In (20), we also introduce the interaction term $|\lambda^1_{t-1} - \lambda^2_{t-1}| \times y^2_t$. Under endogenous group formation, the effect of income changes of agent 2 on agent 1's consumption depends on the initial marginal utility ratio of agent 1 and 2. Broadly speaking, the bigger the difference between initial Pareto weights, the less we would expect income to matter in the endogenous group formation case.

The results are presented in the first four columns of Table 1 and successfully capture the predictions of the two models. The first and third column of Table 1 report the coefficients from regression (19). In the presence of endogenous group formation, the past consumption shares of agent 1 and 2 have significant explanatory power, because they determine the relevant deviation payoff for agent 1. When λ^1_{t-1} is small relative to λ^2_{t-1}, it is optimal to deter a deviation by awarding agent 1 the minimum payoff in a group of size 2. When λ^1_{t-1} is large relative to λ^2_{t-1}, continuation payoffs which give agent 2 the minimum payoff and agent 1 the maximum payoff become optimal. This explains the large negative coefficient on λ^2_{t-1} and the positive coefficient on λ^1_{t-1}. In fact, the effects of the two variables are of very similar magnitude, but go in opposite directions. In contrast, when group size is exogenous, the coefficients on λ^1_{t-1} and λ^2_{t-1} are zero and have no explanatory power because the optimal punishment path always involves reversion to the static autarky equilibrium. The direct effect of agent 2's income is much larger in the case of endogenous group formation because the choice of the optimal punishment path for agent 1 depends on the income realization of agent 2.

In column (2) and (4) of the table we add an interaction effect of $|\lambda^1_{t-1} - \lambda^2_{t-1}| \times y^2_t$ to the specification and restrict the sample to state $S = \{2, 4, 2\}$ and $S' = \{2, 2, 4\}$ in order to keep aggregate income constant. Here, the differential effect of y^2_t in the case of endogenous versus exogenous group formation becomes even more pronounced. In the former scenario, both the direct effect of agent 2's income and the past consumption shares of agent 1 and 2 as well as the interaction effect of these two variables are large and significant. To illustrate the magnitude of the effects, first suppose $\lambda^1_{t-1} = \lambda^2_{t-1}$. Then raising agent 2's income from 2 to 4 increases consumption of agent 1 by 0.37 on average. If we set $|\lambda^1_{t-1} - \lambda^2_{t-1}|$ to its maximum of 0.6, then increasing y^2_t increases the consumption of agent 1 by only 0.056. This is the case because for marginal utility ratios close to 1 it is optimal to increase agent 1's punishment path relative to the case when income realizations of constrained agents are equal in order to compensate for the increased inequality between agents due to agent

	Limited commitment				Limited information	
Consumption of agent 1	Individual deviation		Group deviation			
	(1)	(2)	(3)	(4)	(5)	(6)
λ^1_{t-1}	0.020	-0.003	0.175	0.177	0.861	0.825
	(0.000)	(0.078)	(0.000)	(0.000)	(0.000)	(0.000)
λ^2_{t-1}	0.002	-0.001	-0.138	-0.203	-0.014	-0.055
	(0.184)	(0.347)	(0.000)	(0.000)	(0.131)	(0.337)
y^1_t	0.414		0.401		0.075	-
	(0.000)		(0.000)		(0.000)	
y^2_t	-0.078	-0.025	0.106	0.185	0.002	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.549)	(0.998)
$\lvert\lambda^1_{t-1} - \lambda^2_{t-1}\rvert \times y^2_t$		-0.001		-0.261		0.004
		(0.778)		(0.000)		(0.682)
Y_t	0.182		0.137		0.031	
	(0.000)		(0.000)		(0.000)	

Notes: Data in column (1) and (2) are generated by solving for the equilibrium of a 3-player sub-game perfect limited commitment contract. Data in column (3) and (4) are generated by solving for the equilibrium of a 3-player coalition-proof limited commitment contract. Data in column (5) and (6) are generated by solving for the equilibrium of a 3-player limited information contract. A random error $\epsilon \sim N(0, 0.01)$ is added to each element of the grid of Pareto weights and income realizations. Individual consumption of agent 1 is predicted at 1000 separate grid points where both agent 1 and agent 2 have a binding constraint. P-values in brackets.

Table 1: Predicted consumption when agent 1 and 2 are constrained.

2's higher income realization. However, as $|\lambda^1_{t-1} - \lambda^2_{t-1}|$ increases, the impact of agent 2's income realization diminishes. In the case of exogenous group formation, none of these effects matter.

Finally, we want to use our results to distinguish a limited information environment from a limited commitment environment. As argued in Section 4, the three contracts can be distinguished as follows:

1. History does not matter for constrained agents in the sub-game perfect limited commitment contract.
2. The constrained efficient allocation in the coalition-proof limited commitment contract involves trading off the consumption of all constrained agents.
3. The constrained efficient allocation in a limited information contract involves trading off the consumption of a constrained agent across different states.

Solving the model in Wang (1995) for a group of three agents using the algorithm in Phelan and Townsend (1991), we create a comparable sample of constrained agents in a limited information environment.[5] We then repeat the regressions in (19) and (20) for constrained agents and report the results in column (5) and (6) of Table 1. The results are illuminating and confirm our intuition about the properties of the efficient contract in an imperfect information environment. Firstly, while own past history is highly significant and has a large impact, the past consumption share of other constrained agents has no effect since satisfying the truth-telling constraints does not involve trading off the consumption of all constrained agents. Secondly, while own income has a significant effect on consumption, it is much smaller than in the limited commitment case. This is the case firstly because the deviation payoff does not involve the autarky payoff and secondly because the truth-telling requirement involves cross state constraints. In principle, it is therefore possible to distinguish the two environments empirically.[6]

6 Conclusion

This paper has shown how to model an efficient risk-sharing contract in the presence of endogenous group formation. Requiring groups to be stable with respect to deviations of subgroups as well as individuals alters the predictions of the efficient risk-sharing model substantially. The intervals for next period's marginal utility ratios are history- as well as state-dependent and the covariance between current consumption and past income is no longer constrained to be non-negative. Both these properties allow the researcher to empirically test for the presence of endogenous group formation in informal risk-sharing arrangements using consumption and income data. The predictions also provide scope to distinguish between imperfect commitment versus imperfect information environments.

Establishing the presence or absence of endogenous group formation is an important empirical question, because the impact of policies such as outside financial intermediation and the provision of external safety nets in terms of crowding out of existing arrangements are likely to be very different if insurance group sizes are limited endogenously rather than exogenously. Equally, policy prescriptions will depend on whether risk-sharing is limited by imperfect enforceability or asymmetric information.

[5] We use the same parameters as above to solve the model but impose an exponential utility function. This is necessary, because the truth-telling constraints require that utility is defined for negative values of consumption.

[6] To run this test, we need to be able to identify constrained individuals from observables. As it turns out, regardless of whether the underlying model is one of limited commitment or one of limited information, whether an individual is constrained is highly correlated with having a low consumption share in the previous period and a high income realization in the current period.

Finally, the model in this paper can explain not only the empirical rejection of full insurance, but also why empirical models based on the assumption that risk-sharing arrangements are required to be enforceable with respect to individual deviations may still tend to overestimate the extent of risk-sharing actually observed in the data. In particular, given the nature of enforcement constraints faced by risk-sharing arrangements when contracts are imperfectly enforceable, the model suggests that it is not plausible to expect that idiosyncratic risk can be insured to any significant extent.

7 Appendix A: Proofs

Proof of Lemma 1

1. Define $\widehat{U}_s^n(U_s^1, ..., U_s^{n-1}, \{U_r^1\}_{r=1}^S, ..., \{U_r^{n-1}\}_{r=1}^S) = U^n(\omega, h_t, n)$ such that $U^i(\omega, h_t, n) = U_s^i \quad \forall i = 1, ..., n-1$, where the continuation values $U_r^1, ..., U_r^{n-1}$ satisfy the enforcement constraints in the coalition proof contract. Define the optimal policy correspondence as $G(U_s^1, ..., U_s^{n-1})$. The set of self-enforcing contracts is compact and following from the properties of the utility function, $\widehat{U}_s^n(U_s^1, ..., U_s^{n-1}, \{U_r^1\}_{r=1}^S, ..., \{U_r^{n-1}\}_{r=1}^S)$ is continuous and differentiable and its first derivatives are continuous in both $U_s^1, ..., U_s^{n-1}$ and $\{U_r^1\}_{r=1}^S, ..., \{U_r^{n-1}\}_{r=1}^S$. Hence Corollary 4 and 5 in Milgrom and Segal (2002) apply and the directional derivatives exist and are equal to

$$\begin{aligned} \frac{\partial U_s^{n,+}}{\partial U_s^i} &= -\min \gamma_s^i \\ \frac{\partial U_s^{n,-}}{\partial U_s^i} &= -\max \gamma_s^i. \end{aligned}$$

2. To show that the derivative exists on the range of the optimal policy correspondence G, we follow Cotter and Park (2006) by inspecting the first order condition. Suppose that $U_s^1, ..., U_s^n \in G$ for some initial promised utilities. Then the first order conditions imply that

$$\begin{aligned} \frac{\partial U_r^{n,+}}{\partial U_r^i} + \gamma_r^i &\leq 0 \\ \frac{\partial U_r^{n,-}}{\partial U_r^i} + \gamma_r^i &\geq 0. \end{aligned} \tag{21}$$

Hence, $\frac{\partial U_s^{n,+}}{\partial U_s^i} = \frac{\partial U_s^{n,-}}{\partial U_s^i}$ on the range of the optimal policy correspondence.

3. The derivative of the Pareto frontier is therefore equal to

$$-\frac{\partial U_r^n}{\partial U_r^i} = \gamma_r^i = \frac{u'(c_r^n)}{u'(c_r^i)}. \tag{22}$$

It then follows from the concavity of the utility function that $\frac{\partial U_r^n}{\partial U_r^i} > \frac{\partial U_r^n}{\partial U_r^{i'}}$ if $U_r^{i'} > U_r^i$.■

Proof of Proposition 1:

1) Suppose $\phi_r^i > 0$ for $i = 1, ..., n-1$ and $\phi_r^n = 0$. As a first step, note that (6) is equivalent to a social planner maximizing a weighted sum of expected discounted utilities of n agents. The planner takes as given the planning weights $\{\lambda^i\}_{i=1}^n$, which are normalized to sum to unity and are related to the Lagrange multipliers on the promise keeping constraints in (7) by $\frac{\lambda^i}{\lambda^n} = \gamma^i$. He then chooses consumption c_s^i, promised continuation values U_r^i and the punishment path $\widetilde{V}$ subject to satisfying the enforcement constraints (8)–(9):

$$\begin{aligned} U_s^P &= \max_{((U_r^i)_{r=1}^S)_{i=1}^{n-1}, (c_s^i)_{i=1}^n} \sum_{i=1}^{n-1} \lambda^i \big(u(c_s^i) + \beta \sum_{r=1}^S \pi_r U_r^i\big) \\ &\quad + \lambda^n \big(u(c_s^n) + \beta \sum_{r=1}^S \pi_r U_r^n \big(U_r^1, ..., U_r^{n-1}\big)\big). \end{aligned} \tag{23}$$

Suppose $\phi_r^i > 0$ for $i = 1, ..., n-1$ and $\phi_r^n = 0$. The first order condition with respect to U_r^i is then

$$\begin{aligned} & -\frac{\partial U_r^n}{\partial U_r^i} = \frac{\lambda^i}{\lambda^n}(1 + \phi_r^i) \\ \Leftrightarrow \quad & -\frac{\partial U_r^n}{\partial U_r^i} > \gamma^i \end{aligned} \tag{24}$$

i.e the marginal gain from increasing U_r^i to satisfy the enforcement constraints is less than the marginal loss in utility due to decreasing $U_r^n(U_r^1, ..., U_r^{n-1})$. Therefore, the planner will increase $\{U_r^i\}_{i=1}^{n-1}$ by as little as possible to satisfy the enforcement constraints. In the sub-game perfect contract, this implies choosing the punishment path $\widetilde{V}^i = V^*(1)$ and setting $U_r^i = u(y_r^i) + \beta V^*(1)$, since this is the payoff that agents can achieve by unilateral deviation. Because the autarky payoff is fully determined by the current income realization of agent i, it corresponds to a static equilibrium and is not history-dependent.

2) Suppose that $\phi_r^i > 0$ for $i = 1, ..., n-1$ and $\phi_r^n = 0$. In the coalition-proof contract, the planner must set U_r^i so that $i = 1, ..., n-1$ do not find a joint deviation profitable, and so for any stable subgroup of size m, the punishment path must not lie below the Pareto frontier of that subgroup.

However, from (24) any redistribution from n to those with a binding constraint leads to a fall in total utility. Hence, the punishment path will lie on and not above the Pareto frontier.

Finally, we show that the punishment path $\widetilde{V}$ is history-dependent in the coalition-proof efficient contract. In what follows, a superscript fb indicates the first best contract, whereas a superscript c denotes the constrained contract. The total loss in utility U_s^P when enforcement constraints are binding can be approximated by

$$\begin{aligned} dU_s^P = U_s^{P,fb} - U_s^{P,c} &\approx \beta\pi_r \sum_{i=1}^{n-1} (\lambda^i + \lambda^n \frac{\partial U_r^n}{\partial U_r^i})\Big|_{U_r^{i,c}} (U_r^{i,fb} - U_r^{i,c}) \\ &= \beta\lambda^n \pi_r \sum_{i=1}^{n-1} (\gamma^i - \gamma_r^i)(U_r^{i,fb} - U_r^{i,c}) = -\beta\pi_r \sum_{i=1}^{n-1} \lambda^i \phi_r^i (U_r^{i,fb} - U_r^{i,c}). \end{aligned} \tag{25}$$

From (24), each term in (25) is positive and so the planner will choose $\widetilde{V}$ such that $\lambda^i \phi_r^i (U_r^{i,fb} - U_r^{i,c})$ is as close to zero as possible for all i with a binding constraint. Suppose the largest stable subgroup is of size 2 and agent 1 and 2 have $y_r^1 = y_r^2$ and are the only agents who have a binding constraint. The planner has chosen a particular punishment path $\widetilde{V}$ on the Pareto frontier of a risk-sharing group of size 2. By the symmetry of the stable payoff vectors, for a given total utility $V^1 + V^2$, the planner can either pick a punishment path, such that $V^1 = V^h \geq V^l = V^2$ or $V^2 = V^h \geq V^1 = V^l$, where h and l denote high and low. Also write $U_r^h = u(y_r^1) + \beta V^h$, $U_r^l = u(y_r^1) + \beta V^l$. Then if $\lambda^1 > \lambda^2 \Rightarrow dU^P(V^h, V^l) \leq dU^P(V^l, V^h)$. That is the fall in total utility is less when $U_r^1 = U_r^h$ and $U_r^2 = U_r^l$. Towards a contradiction, suppose

$$\begin{aligned} dU_s^P(V^h, V^l) &\approx \beta\pi_r \big[(\lambda^1 + \lambda^n \frac{\partial U_r^n}{\partial U_r^h}) dU_r^1 + (\lambda^2 + \lambda^n \frac{\partial U_r^n}{\partial U_r^l}) dU_r^2 + \sum_{i=3}^{n-1} (\lambda^i + \lambda^n \frac{\partial U_r^n}{\partial U_r^i}) dU_r^i \big] > \\ dU_s^P(V^l, V^h) &\approx \beta\pi_r \big[(\lambda^1 + \lambda^n \frac{\partial U_r^n}{\partial U_r^l}) dU_r^1 + (\lambda^2 + \lambda^n \frac{\partial U_r^n}{\partial U_r^h}) dU_r^2 + \sum_{i=3}^{n-1} (\lambda^i + \lambda^n \frac{\partial U_r^n}{\partial U_r^i}) dU_r^i \big]. \end{aligned} \tag{26}$$

From (26), it follows that

$$\begin{aligned} & (\lambda^1 + \lambda^n \tfrac{\partial U_r^n}{\partial U_r^h})(U_r^{1,fb} - U_r^h) + (\lambda^2 + \lambda^n \tfrac{\partial U_r^n}{\partial U_r^l})(U_r^{2,fb} - U_r^l) \quad > \\ & (\lambda^1 + \lambda^n \tfrac{\partial U_r^n}{\partial U_r^l})(U_r^{1,fb} - U_r^l) + (\lambda^2 + \lambda^n \tfrac{\partial U_r^n}{\partial U_r^h})(U_r^{2,fb} - U_r^h) \\ \Leftrightarrow \quad & (\lambda^1 - \lambda^2)(U_r^l - U_r^h) + \lambda^n (\tfrac{\partial U_r^n}{\partial U_r^h} - \tfrac{\partial U_r^n}{\partial U_r^l})(U_r^{1,fb} - U_r^{2,fb}) \quad > \quad 0, \end{aligned} \tag{27}$$

which is a contradiction, because the first term is negative by assumption, and the second term is negative because $\frac{\partial U_r^n}{\partial U_r^h} < \frac{\partial U_r^n}{\partial U_r^l}$ from Lemma 1 and the fact that $U_r^{1,fb} \geq U_r^{2,fb}$ when $\lambda^1 > \lambda^2$ and constraints are not binding.

Finally, we show that V^h is strictly greater than V^l when $\lambda^1 > \lambda^2$. Since $\widetilde{V}$ is a pair of utilities on the Pareto frontier of a group of size 2, we can write V^2 as a function of V^1. The difference in total utility between choosing $V^2(V^1) = V^1 = \bar{V}$ and $V^1 = \bar{V} + \epsilon$ and $V^2(\bar{V} + \epsilon)$ is given by

$$\begin{aligned} & U_s^P(\bar{V} + \epsilon, V^2(\bar{V} + \epsilon)) - U_s^P(\bar{V}, V^2(\bar{V})) \approx \\ & \qquad \beta\pi_r\epsilon\Big[\Big(\lambda^1 + \lambda^n \frac{\partial U_r^n}{\partial V^1}\Big|_{V^1=\bar{V}}\Big) + \frac{\partial V^2}{\partial V^1}\Big|_{V^1=\bar{V}}\Big(\lambda^2 + \lambda^n \frac{\partial U_r^n}{\partial V^2}\Big|_{V^2=\bar{V}}\Big)\Big] \end{aligned} \quad (28)$$

Since $\frac{\partial V^2}{\partial V^1}\Big|_{V^1=\bar{V}} = -1$, the above expression is greater zero when $\lambda^1 > \lambda^2$. Therefore, $\lambda^1 > \lambda^2$ and $U_s^1 > U_s^2$ implies that $\widetilde{V}^1 > \widetilde{V}^2$. If $y_r^1 = y_r^2$, this necessarily implies that $U_r^1 > U_r^2$ in the coalition-proof contract. The same argument can be made for any stable subgroup $m = 1, ..., n-1$. Therefore the coalition-proof efficient contract is history-dependent when enforcement constraints are binding.

3) This follows trivially from the fact that the punishment path in a sub-game perfect efficient contract involves reversion to the autarky payoff, which in state r is given by $u(y_r^i) + \beta V^*(1)$.

4) Again, we consider the potential deviation of a coalition consisting of two agents in a group of n agents. We compare the case where both agents have a low income realization, $y_r^1 = y_r^2 = y_l$ to the case in which agent 2 has a higher income realization, $y_r^2 = y_h > y_r^1 = y_l$ and let $\lambda^2 > \lambda^1$. We know from 2) that the optimal choice in the first case implies that $U_r^1 = u(y_l) + \beta\widetilde{V}^l < U_r^2 = u(y_l) + \beta\widetilde{V}^h$, where $\widetilde{V}^l < \widetilde{V}^h$ implies that agent 2's Pareto weight from next period onwards is raised relative to agent 1's Pareto weight. Because the set of punishment paths is symmetric, $\{V^1, V^2\} = \{\widetilde{V}^h, \widetilde{V}^l\}$ is also a feasible punishment path. Define $U_r^{1'} = (1-\alpha)[u(y_l) + \beta\widetilde{V}^l] + \alpha[u(y_h) + \beta\widetilde{V}^h]$ and $U_r^{2'} = (1-\alpha)[u(y_h) + \beta\widetilde{V}^h] + \alpha[u(y_l) + \beta\widetilde{V}^l]$. Now, letting $\alpha = \frac{\beta(\widetilde{V}^h - \widetilde{V}^l)}{u(y_h) - u(y_l) + \beta(\widetilde{V}^h - \widetilde{V}^l)}$, it follows that $U_r^{1'} = u(y_l) + \beta V^h$ and $U^{2'}_r = u(y_h) + \beta V^l$. Then by the concavity of the Pareto frontier on the range of the optimal policy function and the fact that $U_r^n(u(y_l) + \beta\widetilde{V}^l, u(y_h) + \beta\widetilde{V}^h, ..., U_r^{n-1}) = U_r^n(u(y_h) + \beta\widetilde{V}^h, u(y_l) + \beta\widetilde{V}^l, ..., U_r^{n-1})$, we have $U_r^n(U_r^{1'}, U_r^{2'}, ..., U_r^{n-1}) > U_r^n(U_r^1, U_r^2, ..., U_r^{n-1})$.[7] The change in the planner's value function in this case is given by

$$\begin{aligned} U_s^P(U_r^{1'}, U_r^{2'}) - U_s^P(U_r^1, U_r^2) &= \beta\pi_r\big[\lambda^1(\alpha(U_r^2 - U_r^1)) + \lambda^2(\alpha(U_r^1 - U_r^2)) \\ &\quad + \lambda^n(U_r^n(U_r^{1'}, U_r^{2'}, ..., U_r^{n-1}) - U_r^n(U_r^1, U_r^2, ..., U_r^{n-1}))\big]. \end{aligned}$$

[7] Strictly speaking we have only shown that the first partial derivative of the Pareto frontier is decreasing. This is a necessary but not a sufficient condition for concavity. Simulation results suggest that the Pareto frontier is indeed concave.

If $\lambda^1 = \lambda^2$, then $U_s^P(U_r^{1'}, U_r^{2'}) > U_s^P(U_r^1, U_r^2)$. Hence it follows from the continuity of the value function that $\exists\lambda^2 > \lambda^1$, such that $U_s^P(U_r^{1'}, U_r^{2'}) > U_s^P(U_r^1, U_r^2)$, but then $\{U_r^1, U_r^2\} = \{u(y_l) + \beta\widetilde{V}^l, u(y_h) + \beta\widetilde{V}^h\}$ cannot be an optimal choice on the range of deviation payoffs induced by $y_r^2 = y_h > y_r^1 = y_l$. ■

Proof of Proposition 2:

The covariance between own current consumption and previous period income is derived from the first-order conditions (11)–(14). Consumption in period t is determined by the previous period's vector of marginal utility ratios $\gamma = \{\gamma^1, ..., \gamma^{n-1}\}$ and the current income realizations of the n agents in the risk-sharing arrangement:

$$c_t^i = f((y_t^j)_{j=1}^{n-1}, Y_t, \gamma_{t-1}). \quad \forall i = 1, ..., n. \tag{29}$$

Invoking the envelope condition (13), this can be written as

$$\begin{aligned} c_t^i &= f((y_t^j)_{j=1}^{n-1}, Y_t, (U_{t-1}^j)_{j=1}^{n-1}) \\ &= f((y_t^j)_{j=1}^{n-1}, Y_t, d^1((y_{t-1}^j)_{j=1}^{n-1}, Y_{t-1}, (U_{t-2}^j)_{j=1}^{n-1}), ..., d^{n-1}((y_{t-1}^j)_{j=1}^{n-1}, Y_{t-1}, (U_{t-2}^j)_{j=1}^{n-1})) \quad \forall i = 1, ..., n. \end{aligned} \tag{30}$$

Hence, the effect of a change in y_{t-1}^i on c_t^i is given by

$$\frac{dc_t^i}{dy_{t-1}^i} = \sum_{j=1}^{n-1} \frac{\partial c_t^i}{\partial U_{t-1}^j} \frac{\partial U_{t-1}^j}{\partial y_{t-1}^i} \tag{31}$$

In the case of a sub-game perfect efficient contract, all the terms in (31) are positive. Therefore,

$$\text{Cov}(c_t^i, y_{t-1}^i | (y_t^j)_{j=1}^{n-1}, Y_t, (y_{t-1}^j)_{j\neq i}^{n-1}, Y_{t-1}, (U_{t-2}^j)_{j=1}^{n-1}) \geq 0 \tag{32}$$

in a sub-game perfect efficient contract.

In the coalition-proof contract, (32) can be negative by the following argument. Suppose there are two income states and agent 2 finds it profitable to deviate either individually or jointly with agent 1 in both states. If agent 1 has a low income realization, he does not find a deviation profitable, but moving to the high income state he does. Now the planner has to deter a joint deviation, not just an individual one, which means that in such a configuration agent 2 is at least as well off (if not better) under the high income realization as under the low income realization for agent 1. Therefore $\frac{\partial U^j_{t-1}}{\partial y^i_{t-1}} \geq 0$ and $\frac{\partial c^i_t}{\partial U^j_{t-1}} \frac{\partial U^j_{t-1}}{\partial y^i_{t-1}} \leq 0$. If this latter effect outweighs the positive terms in (31), then current consumption may decrease with changes in past income realizations. It follows that

$$\text{Cov}(c^i_t, y^i_{t-1} | (y^j_t)^{n-1}_{j=1}, Y_t, (y^j_{t-1})^{n-1}_{j \neq i}, Y_{t-1}, (U^j_{t-2})^{n-1}_{j=1}) \gtreqless 0 \tag{33}$$

in a coalition-proof efficient contract.■

References

Abreu, D., 1988. On the Theory of Infinitely Repeated Games with Discounting. Econometrica 56, 383–396.

Abreu, D., Pearce, D., Stacchetti, E., 1990. Toward a Theory of Discounted Repeated Games with Imperfect Monitoring. Econometrica 58, 1041–1063.

Ahn, T. K., Isaac, R. M., Salmon, T. C., 2006. Endogenous Group Formation, Mimeo, Florida State University.

Attanasio, O., Rios-Rull, J.-V., 2000. Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare? European Economic Review 44, 1225–1258.

Bernheim, B. D., Peleg, B., Whinston, M., 1987. Coalition Proof Nash Equilibria. I: Concepts. Journal of Economic Theory 42, 1–12.

Bernheim, B. D., Ray, D., 1989. Collective Dynamic Consistency in Repeated Games. Games and Economic Behavior 1, 295–326.

Chaudhuri, A., Gangadharan, L., Maitra, P., 2005. An Experimental Analysis of Group Size and Risk Sharing, The University of Melbourne, Department of Economics, Research Paper 955.

Cotter, K., Park, J.-H., 2006. Non Concave Dynamic Programming. Economic Letters 90, 141–146.

Deaton, A., 1992. Household savings in LDCs: Credit Markets, Insurance and Welfare. Scandinavian Journal of Economics 96, 253–273.

De Weerdt, J., Dercon, S., 2006. Risk-Sharing Networks and Insurance against Illness. Journal of Development Economics 81, 337–356.

Dercon, S., Krishnan, P., 2000. In Sickness and in Health: Risk Sharing within Households in Rural Ethiopia. Journal of Political Economy 108, 688–727.

Dubois, P., 2005. Heterogeneity of Preferences, Limited Commitment and Coalitions: Empirical Evidence on the Limits to Risk Sharing in Rural Pakistan, Mimeo, University of Toulouse.

Fafchamps, M., 1999. Risk Sharing and Quasi-Credit. Journal of International Trade and Economic Development 8, 257–278.

Fafchamps, M., Lund, S., 2003. Risk-Sharing Networks in Rural Philippines. Journal of Development Economics 71, 261–287.

Genicot, G., Ray, D., 2003. Group Formation in Risk-Sharing Arrangements. Review of Economic Studies 70, 87–113.

Grimard, F., 1997. Household Consumption Smoothing through Ethnic Ties: Evidence from Cote d'Ivoire. Journal of Development Economics 53, 391–422.

Judd, K., 1992. Projection Methods for Solving Aggregate Growth Models. Journal of Economic Theory 58, 410–452.

Judd, K. L., 1998. Numerical Methods in Economics. MIT Press, Cambridge, Massachusetts.

Kocherlakota, N. R., 1996. Implications of Efficient Risk Sharing without Commitment. Review of Economic Studies 63, 595–609.

Ligon, E., Thomas, J., Worrall, T., 2002. Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies. Review of Economic Studies 69, 209–244.

Marcet, A., Marimon, R., 1999. Recursive Contracts, Mimeo, Universitat Pompeu Fabra.

Mazzocco, M., 2007. Household Intertemporal Behaviour: A Collective Characterization and a Test of Commitment. Review of Economic Studies 74, 857895.

McGrattan, E. R., 1996. Solving the Stochastic Growth Model with a Finite Element Method. Journal of Economic Dynamics and Control 20, 19–42.

Milgrom, P., Segal, I., 2002. Envelope Theorems For Arbitrary Choice Sets. Econometrica 70, 583–601.

Morduch, J., 1991. Consumption Smoothing Across Space: Tests for Village-Level Responses to Risk, Mimeo, Harvard University.

Murgai, R., Winters, P., Sadoulet, E., de Janvry, A., 2002. Localized and Incomplete Mutual Insurance. Journal of Development Economics 67, 245–274.

Phelan, C., Townsend, R. M., 1991. Computing Multi-period, Information Constrained Optima. Review of Economic Studies 58, 853–881.

Thomas, J., Worrall, T., 1988. Self-Enforcing Wage Contracts. Review of Economic Studies 55, 541–554.

Townsend, R. M., 1994. Risk and Insurance in Village India. Econometrica 62, 539–591.

Udry, C., 1994. Risk and Insurance in a Rural Credit Market: An Empirical Investigation in Northern Nigeria. Review of Economic Studies 61, 495–526.

Wang, C., 1995. Dynamic Insurance with Private Information and Balanced Budgets. Review of Economic Studies 62, 577–595.