

Informal Insurance with Endogeneous Group Size

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Abstract

In this paper, we build a theoretical model where rural communities, in the absence of insurance markets, provide insurance. The extent of risk sharing and the community size are endogenously determined by participation constraints and a collective choice mechanism. Only individual participation constraints and no deviation by coalitions are considered here. Agents with heterogenous risk aversions have conflictual interests both about the membership of the insurance group and about the contract that will be obeyed within the group. A tension exists between the benefits of excluding a marginal member of the group and the costs of exclusion. We stress several dangers for the community structure. An exogenous decrease in income variance, for example, leads to more exclusion. The same holds with an exogenous decrease in mean income.

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1 Introduction

Agricultural activities are at least as risky for rural households in developing countries as anywhere else. This risk is however made much harder to cope with by the lack of markets and of State intervention that constitutes a defining characteristic of rural areas in developing countries. If insurance is to take place at the level of rural communities, it has to be privately provided by the households themselves under the form of a communal institution. There will be no legal framework to enforce the agreement struck within the community, so that the behaviours thereby prescribed have to be self-enforceable.

The theoretical literature on informal insurance has explicitly taken into account the constraint that risk sharing agreements must be self-enforcing. The pioneer paper by Coate & Ravallion (1993) opened the way for a stream of models on informal insurance, typically under moral hazard. The participation constraint put forward by Coate & Ravallion (1993) states that, knowing one's own realization of risk in the current period but ignoring the others' (i.e. at the "ad interim" stage), an agent must prefer complying with the transfer prescribed by the insurance mechanism rather than keeping one's income and hence being excluded from the mechanism and losing the benefits of future insurance. This constraint limits of course the extent of risk sharing that is sustainable within the agreement. The space of possible contracts considered by Coate & Ravallion (1993) has been expanded in Thomas and Worrall (2002), but the crucial and detrimental role of ad interim participation constraints has been confirmed. Genicot (2006) extends the role of moral hazard to preference over partners in a bilateral insurance relationship. The paper shows that, under decreasing absolute risk-aversion, a poorer partner is preferred because he will transfer more in exchange of anticipated insurance benefits. However, all these papers consider bilateral insurance relationships while issues on group formation have received limited attention.

Empirical insights by Goldstein et al. (2005) have stressed another feature of informal insurance schemes : not only they provide incomplete insurance to their members, but they also exclude a fraction of potential participants. Only few theoretical papers have studied the endogenous formation of insurance groups. Relying upon an argument of coalition-proofness, Genicot & Ray (2003) and Genicot & Ray (2005) prove the existence of an upper bound on the size of insurance groups when agents are homogenous and no collective choice mechanism bears upon the extent of risk sharing, determined by the participation constraint (and representative utility maximization when applicable).

We fill in the gap by building an analysis of informal insurance where the extent of risk sharing and the community size are endogenously determined by participation constraints and a collective choice mechanism. Only individual participation constraints and no deviation

by coalitions are considered here. Agents with heterogenous risk aversions have conflictual interests both about the membership of the insurance group and about the contract that will be obeyed within the group. A tension exists between the benefits of excluding a marginal member of the group (i.e. relaxing the participation constraint and allowing more transfers) and the costs of exclusion (i.e. a bigger variance of aggregate income).

Identifying agents subject to exclusion is of particular relevance for policy purposes. Firstly, these people left out informal insurance are the one who must be targeted by scarce State resources and NGO interventions. Secondly, policies affecting income variance and other crucial parameters will influence the membership of insurance groups, leaving more or fewer people unattended. The model presented in the next section enables us to address at the same time welfare issues about excluded agents and positive predictions about the impact of policies on insurance groups.

In section 2, the model is outlined and equilibrium is computed. Comparative statics are performed in section 3 and section 4 concludes.

2 Model

The economy is composed of a finite number of agents i where $i \in \{1, \dots, N\}$ with $N \geq 2$. Each agent lives for an infinity of periods $t \in \{1, 2, \dots, +\infty\}$ and produces a random income Y_{it} that is high h with probability p and low l with probability $(1 - p)$. Y_{it} is independently and identically distributed across agents and periods. In an insurance scheme, two possible problems of moral hazard may arise. The first one is an ex ante moral hazard: agents reduce their effort of having a high income when they are better insured. In this model, we don't account for such an impact in order to focus on the second type of moral hazard. This second type of moral hazard takes place at the ad interim stage: when an agent's income is known, she may prefer to leave the agreement. Agents face the same income distribution because their effort is supposed to be constant. Yet, agents differ in their aversion to risk. On the one hand, a lower risk aversion might originate from a better endowment, hence a lower dependence on fluctuating income. On the other hand, even in very egalitarian societies, households differ in their ability to cope with risk because their access to diversification strategies or their intrinsic preferences towards risk are heterogeneous. In this model, agents' coefficient of risk aversion is uniformly distributed on the interval $[a^-, a^+]$. Agents are sorted by decreasing order of risk aversion, so that for all i , agent i 's coefficient of risk aversion is

$$a_i = a^- + \frac{N - i}{N - 1}(a^+ - a^-). \quad (1)$$

Agents choose whether or not to take part in a mutual insurance agreement. This group has an endogeneous size $n \leq N$. The insurance mechanism works as follows: at each period, each member i concedes a proportion $\alpha \in [0, 1]$ of his income $Y_i \in \{l, h\}$ and the sum of all contributions is equally split. Without loss of generality, this technology is assumed costless. If agents choose not to respect the agreement, once their income level has been revealed, they will be excluded from all future income sharings. Agents have two possible strategies $s \in \{A, O\}$: namely to participate to the mutual insurance agreement, A , or not O .

If she is not part of the mutual insurance agreement, any agent's one-period consumption is $C(O) = Y^1$. For all $i \in \{1, \dots, N\}$ and $t \in \{1, 2, \dots, +\infty\}$, mean and variance are in this case respectively

$$M(O) = ph + (1 - p)l \equiv \mu, \quad (2)$$

$$V(O) = p(1 - p)(h - l)^2 \equiv \sigma^2. \quad (3)$$

If agent i is part of the mutual agreement, her one-period consumption is

$$C_i(A) = (1 - \alpha)Y_i + \alpha \sum_{j=1}^n \frac{Y_j}{n} = Y_i \left(1 - \alpha \frac{(n-1)}{n}\right) + \alpha \sum_{j \neq i} \frac{Y_j}{n},$$

where $i, j \in \{1, \dots, n\}$. The mean income under mutual insurance is unchanged, while the variance of income is smaller:

$$M(A) = ph + (1 - p)l \equiv \mu$$

$$V(A) = \sigma^2 \left(1 - \frac{(n-1)}{n} \alpha(2 - \alpha)\right)$$

Note that for any $\alpha \in [0, 1]$, the larger the community in which informal transfers take place, the lower the variance. This stems from the fact that in the insured income, $\alpha \sum_{j=1}^n \frac{Y_j}{n}$ is less variable when the community size increases.

Agents' instantaneous preferences are defined over the mean and the variance of their income. Agent i at period t has instantaneous utility²:

$$u_{it} = \mu - a_i V(s). \quad (4)$$

¹To make notations shorter and thanks to the i.i.d. assumption on incomes, we write the expressions on incomes without agent and time subscripts.

²Note that one can interpret this utility function in the following way. Preferences are defined over two perfect substitutes, namely the mean and the variance of the agent's income. The marginal rate of substitution between these two goods equals the risk aversion coefficient a_i .

Our analysis will focus exclusively on the steady state values of n and α . This means that for any agent, the income means and variances are identical at each period. For each agent i , the utility function of an agent always participating to the informal agreement can be written as:

$$\begin{aligned} U_i &= \sum_{t=0}^{\infty} \delta^t (M(A) - a_i V(A)) \\ &= \frac{1}{1-\delta} \left[\mu - a_i \sigma^2 \left(1 - \frac{(n-1)}{n} \alpha (2-\alpha) \right) \right] \end{aligned} \quad (5)$$

where $\delta \in (0, 1)$ is the discount factor.

Let us now turn to the endogenous determination of α . An equilibrium is characterized by a pair (n^*, α^*) , that is, a size and a sharing rule for the insurance agreement. As is well-known in the social choice literature, agregation of preferences by voting on a two-dimension space is very problematic. Here, the size of the community in which the agreement takes place is defined by the participation constraint while the only remaining choice is the sharing rule. Then, when the impact of α on the endogenous community size is taken into account, a more tractable problem arises: the agregation of preferences over a one-dimensional space. Then, the single-peakedness of individual preferences guarantees that there exists a unique Condorcet winner which is the preferred level of income pooling of the median voter within the community, α_m^* where m is the median-voter. We now proceed to show that individual preferences are single-peaked in this model.

First order conditions give the preferred income pooling level α_i^* for each agent i :

$$\frac{a_i \sigma^2}{1-\delta} \left(2 \frac{(n-1)}{n} (1-\alpha_i^*) + \frac{\partial n}{\partial \alpha} \frac{1}{n^2} \alpha_i^* (2-\alpha_i^*) \right) = 0 \quad (6)$$

Preferences are single-peaked if the second-order conditions hold not only at α_i^* but also for every possible value of α :

$$\frac{a_i \sigma^2}{1-\delta} \left(-2 \frac{(n-1)}{n} + 2 \frac{\partial n}{\partial \alpha} \frac{1}{n^2} (2-2\alpha) + \frac{1}{n^2} \alpha (2-\alpha) \left(\frac{\partial^2 n}{\partial \alpha^2} - \frac{2}{n} \left(\frac{\partial n}{\partial \alpha} \right)^2 \right) \right) < 0 \quad (7)$$

To check if (7) holds, we need to study how the size of the community, n , behaves when the income pooling level varies. Indeed, the sign of the second order condition depends on how the size of the community n , reacts to a change in the proportion of income pooling within the community ($\frac{\partial n}{\partial \alpha}$ and on $\frac{\partial^2 n}{\partial \alpha^2}$).

As already mentionned in equation (1), agents are determined by their risk aversion coefficient a_i . For any level of α , n is defined by individual participation constraints. Since

the insurance scheme cannot be enforced by an external authority, the informal insurance agreement has to be self enforceable. In order to participate, agents must consent to share a proportion α of their income in order to benefit from the scheme in the future not only *ex ante* but also *ad interim* (i.e. after the private revelation of one's own income). The future loss involved in keeping one's whole income has to be larger than the benefit of it:

$$\begin{aligned} & U_i \left(h \left(1 - \alpha \frac{(n-1)}{n} \right) + \alpha \sum_{j \neq i} \frac{Y_j}{n} \right) + \frac{\delta}{1-\delta} \left(\mu - a_i \sigma^2 \left(1 - \frac{(n-1)}{n} \alpha (2-\alpha) \right) \right) \\ & \geq h + \frac{\delta}{1-\delta} (\mu - a_i \sigma^2) \end{aligned}$$

Note that $\sum_{j \neq i} \frac{Y_j}{n-1}$ is a random variable when n is finite. We assume that agents decide to participate to the mechanism and to share a proportion α of their high income h *before* observing the other agents' income. Participation is decided *ad interim*. In this case, they consider the other agents' payoffs in expected terms:

$$\sum_{j \neq i} \frac{Y_j}{n-1} = ph + (1-p)l. \tag{8}$$

If agents were able to observe its realization before contributing, the size of the community would change as time moves forward. A study of the community's dynamics is interesting in itself, but our contribution is aimed at highlighting clearly the driving forces behind group size and insurance level formation. Considering its dynamics would not provide such clear-cut conclusions as those we obtain. Consequently, the equilibrium community size is defined at period 1 and does not change over time. There is at least another way of modeling participation in this respect by assuming that each agent observes all incomes at each period before contributing. In this case, participation is decided *ex post* and we need a further assumption to focus on a steady state for n and α . Because we consider an infinity of periods and because the joint probability that $\sum_{j \neq i} \frac{Y_j}{n-1} = l$ and $Y_i = h$ is quite low but positive, at the steady state, the community will shrink until it reaches the steady-state size defined by $\sum_{j \neq i} \frac{Y_j}{n-1} = l$. Notice that in this way of modeling participation constraints are more binding and lead up to a smaller community size. This may explain why in some areas people hide part of their incomes to relax the participation constraint. Indeed, it is more efficient socially that everyone hide *ad interim* her income so that the community size will be greater and the sharing of income higher.

The ad interim participation constraint (??) may be rewritten as:

$$\begin{aligned}
& h - \alpha \frac{(n-1)}{n} \left[(1-p)(h-l) - \alpha \frac{1}{n} a_i \sigma^2 \right] + \frac{\delta}{1-\delta} \left(\mu - a_i \sigma^2 \left(1 - \frac{(n-1)}{n} \alpha (2-\alpha) \right) \right) \\
& \geq h + \frac{\delta}{1-\delta} (\mu - a_i \sigma^2) \\
& \iff a_i \geq \frac{1}{p(h-l)} \frac{1}{\left(\frac{\delta}{1-\delta} (2-\alpha) - \frac{\alpha}{n} \right)}. \tag{9}
\end{aligned}$$

If the right handside of the inequation is either bigger than a^+ or smaller than a^- then a corner solution arises and the rest of the model is trivial. We therefore focus on interior solutions.

Let us now define the size of the community based on this threshold on risk aversions. Note first that, ignoring integer problems (rounding down n is a natural solution but it would preclude n from being differentiable for no additional insights), the index value defining the last agent for whom this condition is satisfied is precisely the equilibrium value of n :

$$\begin{aligned}
n &= N + \frac{(N-1)}{a^+ - a^-} \left(a^- - \frac{1}{p(h-l)} \frac{1}{\left(\frac{\delta}{1-\delta} (2-\alpha) - \frac{\alpha}{n} \right)} \right) \\
&\iff n - N - \frac{(N-1)}{a^+ - a^-} \left(a^- - \frac{1}{p(h-l)} \frac{1}{\left(\frac{\delta}{1-\delta} (2-\alpha) - \frac{\alpha}{n} \right)} \right). \tag{10}
\end{aligned}$$

The community size has to be greater or equal to 1 and to be smaller than N . This is always the case if $a^- \leq \frac{1}{p(h-l)} \frac{1}{\left(\frac{\delta}{1-\delta} (2-\alpha) - \frac{\alpha}{n} \right)} \leq a^+$.

n directly depends on the level of income pooling applied within the group. $1 + \frac{(N-1)}{a^+ - a^-} \frac{\partial a_i}{\partial n} \geq 0$ is a sufficient condition to insure that n is a decreasing and concave function of α :

$$\begin{aligned}
\partial n / \partial \alpha &= - \frac{\frac{(N-1)}{a^+ - a^-} \frac{\partial a_i}{\partial \alpha}}{1 + \frac{(N-1)}{a^+ - a^-} \frac{\partial a_i}{\partial n}} < 0, \\
\partial^2 n / \partial \alpha^2 &= - \frac{\left[\frac{(N-1)}{a^+ - a^-} \frac{\partial^2 a_i}{\partial \alpha^2} \left(1 + \frac{(N-1)}{a^+ - a^-} \frac{\partial a_i}{\partial n} \right) - \left(\frac{(N-1)}{a^+ - a^-} \right)^2 \frac{\partial a_i}{\partial \alpha} \frac{\partial^2 a_i}{\partial \alpha \partial n} \right]}{\left(1 + \frac{(N-1)}{a^+ - a^-} \frac{\partial a_i}{\partial n} \right)^2} < 0.
\end{aligned}$$

The case in which the sufficient condition to ensure that $\partial n / \partial \alpha$ and $\partial^2 n / \partial \alpha^2$ are negative is the more likely to be binding when $\alpha = 1$ and $n = 2$. In that case, it may be written as $\frac{(N-1)}{a^+ - a^-} \frac{1}{p(h-l)} \frac{(1-\delta)^2}{(3\delta-1)^2} < 1$. This condition is satisfied when agents are sufficiently patient (δ close to 1) or if risk diversification opportunities are scarce ($\frac{N-1}{a^+ - a^-}$ small enough). It means that the model developed in this paper may only be applied in an environment

where agents are sufficiently patient to build a mutual insurance group and where there are few other diversification opportunities to deal with risk.

As a reminder, these derivatives are crucial for the endogenous determination of α . To have single-peaked preferences, we have to check that the second order condition (7) is everywhere negative. This condition depends on the behavior of the community size when the level of income pooling varies. Because the community size decreases when α increases, the condition for single-peaked preferences over levels of α is satisfied. Consequently, the equilibrium value of the share of income pooling, α^* , is given by the implicit function:

$$a_m \frac{\sigma^2}{1 - \delta} \left(2 \frac{(n-1)}{n} (1 - \alpha_m^*) + \frac{\partial n}{\partial \alpha} \frac{1}{n^2} \alpha_m^* (2 - \alpha_m^*) \right) = 0$$

where n is given by an implicit function of α and n (equation (10)) and where a_m is the risk aversion coefficient of the median agent of the community of size n . Thanks to the particular form of the mean-variance utility function, people inside the community will always be agree on the level of income sharing. Indeed, the preferred α doesn't depend on the risk aversion of the agents:

$$2 \frac{(n-1)}{n} (1 - \alpha^*) + \frac{\partial n}{\partial \alpha} \frac{1}{n^2} \alpha^* (2 - \alpha^*) = 0. \quad (11)$$

A striking result of our model is that, when people have mean-variance preferences, they deal with risk by adjusting the community size while the preferred proportion of shared income for a given community size is the same for all agents participating to the insurance scheme. So the aggregation of individual preferences is straightforwardly solved: the unanimously preferred choice within the community is picked.

There exists a unique equilibrium (α^*, n^*) . Indeed, using the *Intermediate Value Theorem*, we may easily show that there always exists an equilibrium with $\alpha \in]0, 1[$ for all possible values of the parameters. Furthermore, $\frac{\partial n}{\partial \alpha} < 0$ and $\frac{\partial \alpha}{\partial n} > 0$ are sufficient to insure that this equilibrium is unique. This value of α^* gives rise to a single value of n comprised between 0 and N .

Remark that if the population size N is very large, there is one equilibrium $(1, \infty)$. That can be noticed by checking that this pair satisfies the equilibrium conditions and because of the uniqueness of the equilibrium. If $N \rightarrow \infty$, the cost to exclude is zero and the community will exclude all agents who will not participate when $\alpha = 1$. There will be full risk sharing and the risk disappears (the size of the community is such that the consumption of the entire community is constant).

Along equation (11), it is also striking that $\frac{\partial \alpha}{\partial n} > 0$ for all possible values of n . The tension between this fact and the equation (10) where $\frac{\partial n}{\partial \alpha} < 0$ is a driving force behind the comparative statics that follows.

2.1 Comparative Statics

2.1.1 Mean Preserving Spread

We want to study an increase in the income variation. Indeed, a higher income risk is expected to have effects both on the insurance agreement within the community and on the community size. In order to study its impact, we will increase the risk and simultaneously keep the income mean constant. Because we have 3 parameters, p, l and h , there are an infinity of manners to model such a mean-preserving spread. One way to do it is to keep h constant and let l and p vary. We may write p and l as functions of μ, σ^2 and h :

$$\begin{cases} \mu = ph + (1-p)l \\ \sigma^2 = p(1-p)(h-l)^2 \end{cases} \iff \begin{cases} l = \mu - \frac{\sigma^2}{h-\mu} \\ p = \frac{\sigma^2}{\sigma^2 + (h-\mu)^2} \end{cases}.$$

We first study how the proportion of income pooling reacts when the risk increases. In order to compute this impact, we use the implicit function theorem on equations (11) and (10):

$$\frac{d\alpha}{d\sigma^2} > 0.$$

The level of income sharing increases when the variance of income is higher. Indeed, the direct effect of a higher variance on the chosen α^* is positive. When the risk increases, all agents want to share more because the cost of sharing is smaller (the participation constraint is less binding while the variance of the income increases). The indirect effect through n is also positive. When σ^2 increases, the risk aversion coefficient, for which the constraint (9) is binding, is lower for a given α^* . Hence, the participation constraint is relaxed by an increase in σ^2 and will be satisfied for smaller values of a_i so that the community size will be bigger. Equation (11) implies that a bigger community will share more. Both effect concur, meaning that when risk increases, people tend to share more inside the community.

We than study the impact of a larger risk on the size of the community within which the insurance agreement takes place:

$$\frac{dn}{d\sigma^2} > 0.$$

When σ^2 increases, the direct effect on the community size is positive because, as we already mentionned above, the participation constraint will be satisfied for smaller values of a_i meaning a bigger community size. This effect always dominates the indirect effect through the income share level within the community. Indeed, α^* will increase because the cost of insurance is less important. The total effect is positive because agents prefer to increase α^*

up to a level where more people will participate to the community with the increased risk. It means that the cost of not deviating from the insurance scheme is always dominated by the benefit of sharing more in a bigger community.

When risk increases, more people are insured and all agents participating to the informal insurance scheme share a higher proportion of income. The impact of a higher risk, in our model, amounts to a reinforcement of local communities.

Notice that the result is robust to a different specification of a mean-preserving spread. Indeed, with a mean-preserving spread, keeping p constant and letting l and h vary, the impacts on α^* and n are the same.

2.1.2 Effect of a wealth variation preserving the income variance

A change in the global wealth of the population keeping the variance of the income constant will affect the equilibrium (α^*, n) . As we already mentionned, with three parameters, p , h and l , there are an infinity of ways to keep σ^2 (or μ) constant while μ (or σ^2) vary. As for the mean-preserving spread, we may express equations (11) and (10) as functions of α^* , μ , σ^2 and h :

$$\begin{cases} 2\frac{(n-1)}{n}(1-\alpha^*) + \frac{\partial n}{\partial \alpha} \frac{1}{n^2} \alpha^*(2-\alpha^*) = 0 \\ n - N - \frac{(N-1)}{a^+ - a^-} \left(a^- - \frac{h-\mu}{\sigma^2} \left(\frac{1}{\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}} \right) \right) = 0. \end{cases}$$

The effect on the level of income pooling of an increase in the income mean, μ , preserving the variance, σ^2 , is

$$\frac{d\alpha^*}{d\mu} > 0.$$

Indeed, both direct and indirect effect are positive. When people become more wealthy, they prefer to share more because the cost of sharing reduces.

We then evaluate this effect on n :

$$\frac{dn}{d\mu} \leq 0,$$

When the wealth increases the direct effect is positive because it relaxes the participation constraint while the indirect effect is negative. The reason is that when the wealth increases the income sharing within the community increases which reinforce the participation constraint and reduces the community size. But $\frac{1}{2} + \frac{(N-1)}{a^+ - a^-} \frac{\partial a_i}{\partial n} \geq 0$ is a sufficient condition to insure that the direct effect always dominate the indirect effect. If the individuals are sufficiently patient or the risk diversification possibilities inside the community are scarce, we are certain that a wealth increase will reinforce the community.

Notice that with a different way of modeling a variation of the income mean preserving the variance, we may have no impact on the equilibrium values of (α^*, n) . Indeed, if we

suppose a simultaneous increase in the high income and in the low income, keeping p and $(h - l)$ constant, the impact on both α^* and n is zero.

2.1.3 Effect of a high income variation

There are two reasons why the variation of the high income is an important topic for study. Firstly, technology adoption will mainly improve the living conditions in good states of nature. For example, fertilizers are not likely to improve subsistence in case of drought but they will enhance production under favourable circumstances. Secondly, the high income state plays the crucial role in the participation constraint because only the agents with a high income may choose to drop off the insurance agreement. Intuitively, one might fear that an improvement of the high income act against the extent of risk sharing and against the participation of a large number of agents to the insurance scheme.

If we allow only h to increase, the impact on α^* and n has the same sign than the one for the mean preserving spread:

$$\begin{aligned} \frac{d\alpha^*}{dh} &> 0, \\ \frac{dn}{dh} &> 0. \end{aligned}$$

Indeed, the effect of a higher h increases the variability of the income. Insurance in the whole population is better: more agents are insured and all agents in the insurance scheme share a larger part of their income.

If a technological change implies a higher income in the case of high income, reinforcement of local communities may occur. It means that all individuals are better off.

2.1.4 Variation in the population heterogeneity

To study an increase in the population heterogeneity we spread the interval of the risk aversion coefficients $[a^-, a^+]$.

The effect on the income pooling level is indeterminate:

$$\frac{d\alpha}{d[a^-, a^+]} \lesseqgtr 0.$$

The direct effect of the interval spread on α is positive. Indeed, increasing $[a^-, a^+]$ reduces the density of population in the interval of risk aversion and it decreases the cost of sharing in terms of number of people who leave the insurance group. The indirect effect is indeterminate. On the one hand, spreading the whole interval $[a^-, a^+]$ mechanically increases n according to

(10). On the other hand, it also spreads the interval $[a^-, a_i]$ (risk aversions of agents outside the community). However, $a_i > \frac{1}{2}(a^+ + a^-)$ is a sufficient condition to insure that the total effect $\frac{d\alpha}{d[a^-, a^+]}$ is positive. Indeed, if the interval $[a^-, a^+]$ spreads, with a same group size n , the mean of risk aversion inside the community composed of the more risk averse agents will increase and as a result, the preferred share of income inside the community increases.

However, the effect of an increase in the population's heterogeneity on the community size cannot be ascertained even with the condition:

$$\frac{dn}{d[a^-, a^+]} \leq 0.$$

It can come as a surprise that despite the decreasing cost of sharing in terms of membership the size of community may decrease as a result of increase heterogeneity in risk aversion. The improved sharing may actually impose more exclusion.

2.1.5 Variation in the population size N

When N increases, the possibilities of risk diversification increase. Indeed, when the population size increases (the interval of risk aversion coefficients $[a^-, a^+]$ remains identical), for a given level of income pooling α , more agents will satisfy the participation constraint and be able to belong to the group. Therefore, it is easy to reduce the variability of a portion of the income by increasing the community size.

The population size N have an ambiguous impact on the level of income pooling:

$$\frac{d\alpha}{dN} \leq 0.$$

The direct effect is negative because when N increases the cost of sharing in terms of membership increases as well. On the other hand, the indirect effect is positive because n is mechanically increasing with N (see equation (10)).

The impact of an increase in the diversification possibilities on the community size is positive:

$$\frac{dn}{dN} > 0.$$

As already mentioned, n is mechanically increasing with N but there is also a twofold effect via the preference for sharing. When N increases for a given a_i , there are more people within the community and hence a better diversification of idiosyncratic risks (the variance of mean income within the community decreases)

3 Conclusion

This paper provides an analysis of informal insurance where the extent of risk sharing and the community size are endogenously determined by participation constraints and a collective choice mechanism. For tractability, a mean-variance utility function was assumed. Agents have heterogeneous risk aversions, giving rise to conflictual interests about the membership of the community and the fraction of income shared. We have shown a tension between the decreasing impact of sharing on the group size, through the marginal agent's participation constraint, and the increasing impact of the group size on sharing, through a sheer risk-aversion effect.

We have also stressed several dangers for the community structure. An exogenous decrease in income variance, for example, leads to more exclusion. The same holds with an exogenous decrease in mean income. Serious worries for rural communities may appear if development policies that reduce risk in agricultural activities are planned.

References

- Ambec, S. (2005). Voting over informal risk sharing rules.
- Coate, S. & Ravallion, M. (1993). Reciprocity without commitment: Characterization and performance of informal insurance arrangements. *Journal of Development Economics*, 44, 1–24.
- Fafchamps, M. (1992). Solidarity network in rural africa: Rational peasant with a moral economy. *Economic Development and Cultural Change*, 41(1), 147–177.
- Fafchamps, M. (1995). *The rural community, mutual assistance, and structural adjustment. From "State, Markets, and Civil Institutions: New Theories, New Practices, and their Implications for Rural Development"*. Mc Qillan Press.
- Fafchamps, M. (2003). *Rural Poverty, Risk and Development*.
- Fafchamps, M. & Lund, S. (2003). Risk-sharing networks in rural philippines. *Journal of Development Economics*, 71, 261–287.
- Genicot, G. & Ray, D. (2003). Endogenous group formation in risk-sharing arrangements. *Review of Economic Studies*, 70.
- Genicot, G. & Ray, D. (2005). *Informal Insurance, Enforcement Constraints, and Group Formation. From "Group Formation in Economics: Networks, Clubs and Coalitions"*. Cambridge University Press.
- Gibbard, A. (1973). Manipulation of voting schemes: a general result. *Econometrica*, 41, 587–601.
- Goldstein, M., de Janvry A., & E., S. (2005). *Is a Friend in Need a Friend Indeed? From "Insurance against Poverty"*. Oxford University Press.
- Hoff, K. & Sen, A. (2005). The kin system as a poverty trap? *World Bank Policy Research Working Papers*, 3575.
- May, K. (1952). A set of independent necessary and sufficient conditions for simple majority decision. *Econometrica*, 20, 680–684.
- Moulin, H. (1988). *Axioms of Cooperative Decision Making*. Cambridge University Press.
- Platteau, J.-P. (2000). *Institutions, Social Norms, and Economic Development*. Amsterdam, Netherland: Harwood Academic Publishers.

- Satterthwaite, M. (1975). Strategy-proofness and arrow's conditions : Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10, 187–217.
- Wade, R. (1994). *Village Republics: Economic Conditions for Collective Action in South India*. San Francisco: ICS Press.
- Wolf, C. (1955). Institutions and economic development. *American Economic Review*, 45 (5), 867–883.