Complementarity, Convexity and the Productive Value of Biodiversity: Evidence from an Agroecosystem in Ethiopia

by

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Abstract: The paper investigates the value of biodiversity as it relates to the productive value of services provided by an ecosystem. It analyzes how the value of an ecosystem can be "greater than the sum of its parts." First, it proposes a general measure of the value of biodiversity. Second, this measure is decomposed into four components, reflecting the role of complementarity, scale, convexity and catalytic effects. This provides new information on the sources and determinants of biodiversity value. Third, the methodology is applied to analyze the productive value of diversity of an agroecosystem in the Highlands of Ethiopia. The analysis provides estimates of the value of diversity and its components. The value of diversity is estimated to be positive. The complementarity component is found to be large and statistically significant: it is the main source of biodiversity value in this agroecosystem of Ethiopia. However, the convexity component is negative, indicating that non-convexity contributes to reducing the value of biodiversity.

Keywords: biodiversity, productive value, complementarity, convexity.

JEL: D6, Q2, Q5

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1. Introduction

In recent years the scientific community has called attention to the unprecedented loss of biodiversity in agroecosystems. These concerns are reflected in the Convention on Biological Biodiversity and the International Treaty on Crop Genetic Resources, which encourage and promote on-farm conservation and biodiversity in agroecosystems. It has been argued that biodiversity is an important component of ecological systems (e.g., Heal; Tilman and Downing; Tilman et al.; Wood and Lenné). There is empirical evidence that a loss of biodiversity can have adverse effects on the functioning of ecosystems (e.g., Loreau and Hector; Naeem et al.; Tilman and Downing; Tillman et al.). Yet, while the term biodiversity has acquired a positive connotation, its measurement and conceptualization remains difficult. In the agroecological literature, explanations for the diversity productivity relationships have included the species complementarity hypothesis and the scale hypothesis (Callaway and Walker; Loreau and Hector; Sala et al.; Tilman et al;).[1] Complementarity in an ecosystem arises when particular species perform better in the presence of others (either because they actively cooperate with each other or because niche partitioning allows them to make better use of available resources). The scale hypothesis reflects the fact that the functioning of an ecosystem can be affected by its size. Agro-ecosystems are composed of many different patch types. The spatial heterogeneity of the patches is thus an important determinant of crop species coexistence and the relation between diversity and productivity (Giller et al.; Tilman and Kareiva).

To measure the role of diversity on productivity, two approaches have appeared in the literature. One approach is based on ecological diversity indices, including the Margalef, the Shannon index and the Simpson index (e.g., Hill; Lande; May; Polasky and Solow; Simpson). These diversity metrics rely on species richness, dominance or abundance. They have been used extensively in the empirical analysis of biodiversity issues (e.g., Di Falco and Chavas; Heisey et al.; Meng et al.; Priestley and Bayles; Smale et al., 1998, 2002, 2003). Yet, the use of diversity indices raises several issues. First, different indices seems to deliver very different results and there is a debate on which diversity index is most appropriate (e.g., Routledge). At this point, it appears that no particular index is always superior. This point is made clear when the value of biodiversity is found to depend on the presence and nature of complementary among ecosystem services (e.g., Faith et al.; Justus and Sarkar; Loreau and Hector). Second, diversity indices do not identify the sources of diversity value. This is problematic to the extent that knowing the source and nature of diversity value is often important in evaluating alternative management strategies for diversity. Addressing these issues appears challenging. The other approach developed by Weitzman (1992, 1998) measures biodiversity through a diversity function based on a measure of dissimilarity. However, Brock and Xepapadeas have shown that a more diverse ecosystem can be much more valuable even when the increase in dissimilarity is almost zero. This reflects the complexity of ecosystems. It also suggests the need for further research on the characterization and valuation of biodiversity.

The objective of this paper is to develop a general analysis of the productive value of biodiversity. The research focuses on the productive services provided by an ecological system. In this context, it relies on a representation of the underlying technology to characterize the value of biodiversity. It applies under general conditions, allowing for non-convexity, lack of free-disposal in environmental goods. It analyzes how the value of an ecosystem can be "greater than the sum of its parts." First, it proposes a general measure of the value of biodiversity that does not rely on any index. Second, it is shown that the value of diversity can be decomposed into four additive parts: one associated with complementarity, one with scale effects, one with convexity effects, and one with catalytic effects. This provides new information on the sources and determinants of biodiversity value. Third, the methodology is applied to analyze the productive value of diversity of an agroecosystem in the Highlands of Ethiopia. The analysis provides estimates of the value of diversity and its components. The value of diversity is estimated to be positive. The complementarity component is found to be large and statistically significant: it is the main source of biodiversity value in this agroecosystem of Ethiopia. However, the convexity component is negative, indicating that non-convexity contributes to reducing the value of biodiversity.

2. The Productive Value of an Ecosystem

Consider an ecological system as a production process involving a set of goods $z = (z_1, z_2, \ldots) \in R^{m+n}$. We use the netput notation where quantities are defined to be negative for inputs (with $z_i \leq 0$ when the i-th netput is an input) and positive for outputs (with $z_i \geq 0$ when the i-th netput is an output). The ecosystem involves two types of netputs: private goods z_a and other goods z_b, with $z = (z_a, z_b)$. We assume that the private goods $z_a = (z_1, \ldots, z_m) \in R^m$ are market goods with prices $p \in R^m_{++}$. The goods $z_b = (z_{m+1}, \ldots, z_{m+n}) \in R^n$ can be non-market goods, i.e. goods without observable market prices. In this section n, it will be convenient to assume that z_b represents n environmental goods. The underlying production technology is denoted by the set $Z \subset R^{m+n}$, where $z \equiv (z_a, z_b) \in Z$ means that private goods z_a can be feasibly produced in the presence of environmental goods z_b. Throughout, we assume that the set Z is closed, and that it exhibits free disposal with respect to the private goods z_a (where free disposal in z_a means that, for any $z \equiv (z_a, z_b) \in Z$, $z_a' \leq z_a$ implies that $(z_a', z_b) \in Z$). However, we do <u>not</u> assume that the set Z is convex, or that it exhibits free disposal with respect to z_b. Thus, our analysis applies under a general technology characterizing the productivity of the ecological system: it allows for <u>non-convexity</u>; and it does not require that <u>the environmental goods z_b exhibit free-disposal</u>.

This is illustrated in Figure 1, where m = 1, n = 1, and the upper bound of the feasible set Z is given by the line ABCDEFGH. In the region EFG both z_a and z_b are considered as outputs. This would apply to a healthy ecosystem that allows for the production of valuable ecological services as well as private goods. In the region BCDE, the environmental good z_b is an input in the production of z_a. This corresponds to situations where the ecological system is used mainly to produce private goods (e.g., agriculture using the ecological system to produce food). Region GH corresponds to a case of environmental enhancement where the private good z_a is an input into the production of the environmental good z_b (e.g., protecting the ecological system that provides clean water for New York City). Finally, in the region AB, both z_a and z_b are inputs. This would correspond to unproductive ecological systems where the production of private goods becomes impossible (e.g., on Mount Everest). Note that the technology Z in Figure 1 is not convex (e.g., the region CDEF). And it does not exhibit free disposal in the environmental good z_b in the regions ABC and DEF.

We are interested in providing a general representation of the frontier technology given by the boundary of Z. Such a representation is given by the shortage function proposed by Luenberger. Let $g \in R^m_+$ be a reference bundle of private goods satisfying $g \geq 0$, and $g \neq 0$. For a given g, the shortage function S(z, g) evaluated at point $z \equiv (z_a, z_b)$ is defined as

$$S(z, g) = \min_\alpha \{\alpha: (z_a - \alpha g, z_b) \in Z\}, \text{ if there is an } \alpha \text{ such that } (z_a - \alpha g, z_b) \in Z\},$$
$$= +\infty \text{ otherwise.} \quad (1)$$

The shortage function S(z, g) measures the number of units of the reference bundle g reflecting the distance between point $z \equiv (z_a, z_b)$ and the frontier technology. It has some useful properties (see Luenberger):

1. $z \in Z$ implies $S(z, g) \leq 0$,
2. Under free disposal in z_a, $Z = \{z: S(z, g) \leq 0\}$,
3. Under free disposal in z_a, $S(z_a, z_b, g)$ is non-decreasing in z_a,
4. $S(z_a + \alpha g, z_b, g) = \alpha + S(z, g)$, for any α.

Property 1 shows that $S(z, g) \leq 0$ is associated with the feasibility of the netputs $z \equiv (z_a, z_b)$. Under free disposal in z_a, property 2 implies that $S(z, g) \leq 0$ provides a complete characterization of the technology. In this case, $S(z, g) = 0$ if and only if z is on the upper bound of the feasible set Z, with $S(z, g) = 0$ providing a multi-input multi-output functional representation of the underlying frontier technology. Under free disposal in z_a, property 3 states that the shortage function $S(z_a, z_b, g)$ is non-decreasing in the private goods z_a. Note that, in general, $S(z_a, z_b, g)$ can be either increasing or decreasing in the environmental goods z_b. As suggested by property 3, it would be non-decreasing in z_b if the technology exhibited free disposal in z_b. But it would be decreasing in z_b in regions where free disposal in

z_b fails to hold. Finally, if $S(z_a, z_b, g)$ is twice differentiable in z, property 4 implies that $\frac{\partial S(z_a, z_b, g)}{\partial z_a}$ g = 1 and that $\frac{\partial^2 S(z_a, z_b, g)}{\partial(z_a, z_b)\partial z_a}$ g = 0.

The shortage function is illustrated in Figure 1. Consider evaluating it at point J, where the private good $z_a > 0$ is an output (represented by the distance OK in Figure 1) and the environmental good $z_b < 0$ is an input (where $|z_b|$ is given by the distance OL). Given the reference bundle g (represented by JM in Figure 1), the shortage function S(z, g) evaluated at point J is given by -JN/JM.

As a further illustration, consider the special case where g = (1, 0, …, 0). Then $S(z, g) = \min_\alpha \{\alpha: (z_1 - \alpha, z_2, \ldots, z_{m+n}) \in Z\} = z_1 - G(z_c)$ where $z_c = (z_2, \ldots, z_{m+n})$ and $G(z_c) = \max \{z_1: (z_1, z_c) \in Z\}$ is the largest possible z_1 that can be obtained given other netputs z_c. When z_1 is an output, $G(z_c)$ is a standard production function representing the underlying technology, where feasibility is given by $z_1 \leq G(z_c)$. In this case, under differentiability, $\partial S/\partial z_1 = 1$ and $\partial S/\partial z_c = -\partial G/\partial z_c$, implying that $-\partial S/\partial z_c$ can be interpreted as measuring the marginal product of z_c.

For a given $z \equiv (z_a, z_b)$, the shortage function S(z, g) in (1) provides a convenient basis for analyzing the productive value of the environmental goods z_b. To see that, consider a change in environmental goods from z_b^1 to z_b^2. Then, define

$$P(z_a, z_b^1, z_b^2, g) = S(z_a, z_b^1, g) - S(z_a, z_b^2, g). \tag{2}$$

Starting from the point $z \equiv (z_a, z_b^1)$, $P(z_a, z_b^1, z_b^2, g)$ in (2) measures the number of additional units of the reference bundle g that can be obtained from changing environmental goods from z_b^1 to z_b^2. To illustrate, consider the case where z_b are inputs (with $z_b < 0$) and (2) is evaluated under a technology exhibiting free disposal in z_b. As suggested by property 3, $S(z_a, z_b, g)$ would be non-decreasing in z_b. Then, with $z_b < 0$, any increase in the environmental inputs from $|z_b^1|$ to $|z_b^2|$ would mean a decrease in z_b, implying that $P(z_a, z_b^1, z_b^2, g) \geq 0$ in (2). In this case, increasing environmental input z_b can make it possible to produce more of the private goods z_a, with $P(z_a, z_b^1, z_b^2, g) \geq 0$ measuring the additional number of units of the private goods g that can be produced.

To note the role of free disposal for the environmental goods z_b, consider the case of an increase in the environmental input from point J in Figure 1. With $z_b < 0$, increasing the environmental input $|z_b|$ means a decrease in z_b from point J, implying an increase in the shortage function. This reflects the fact that free disposal in z_b does not hold in the region BC of Figure 1, and that the shortage function $S(z_a, z_b, g)$ is now decreasing in z_b in the neighborhood of point J. In this case, any increase in the environmental input $|z_b|$ implies that $P(z_a, z_b^1, z_b^2, g) < 0$ in (2). This illustrates that, without free disposal, increasing environmental input z_b can reduce the ability to produce the private goods z_a, with $P(z_a, z_b^1, z_b^2, g) < 0$ measuring the associated reduction in the number of units of g that can be produced.

In the case where the private goods z_a are market goods with prices p, a monetary evaluation of $P(z_a, z_b^1, z_b^2, g)$ in (2) is

$$V(z_a, z_b^1, z_b^2, p, g) = P(z_a, z_b^1, z_b^2, g)\,(p\, g)$$

$$= [S(z_a, z_b^1, g) - S(z_a, z_b^2, g)]\,(p\, g). \quad (3)$$

Starting from the point $z \equiv (z_a, z_b^1)$, $V(z_a, z_b^1, z_b^2, p, g)$ in (3) gives a monetary value of the private goods that can be obtained when environmental goods change from z_b^1 to z_b^2. Then, comparing (2) and (3) gives the following result.

Proposition 1: When the reference bundle g is chosen to have unit value (with $p\, g = 1$), $P(z_a, z_b^1, z_b^2, g)$ in (2) gives a monetary value of changes in environmental goods from z_b^1 to z_b^2.

This provides some guidance for choosing the reference bundle g. When g is chosen such that $p\, g = 1$, Proposition 1 shows that $P(z_a, z_b^1, z_b^2, g)$ in (2) measures the monetary value of changes in environmental goods. This measure is attractive on several grounds: it allows the analysis of environmental goods as "non-market goods", (i.e., goods with no observable price); it allows for a general technology underlying the productivity implications of an ecological system; it does not require the technology to be convex; and it does not require that the environmental goods satisfy "free disposal."

As shown in Proposition 1, equations (2) and (3) provide absolute measures of changes environmental goods. Note that these measures can be easily modified into relative measures. To see that, consider the case where $z_b^1 = 0$ and $z_b^2 = z_b$. Then, equation (2) becomes

$$P(z_a, 0, z_b, g) = S(z_a, 0, g) - S(z_a, z_b, g), \tag{2'}$$

where $P(z_a, 0, z_b, g)$ measures the total value of the environmental goods z_b when $p\ g = 1$. In situations where $P(z_a, 0, z_b, g) \neq 0$, a relative measure of changes in environmental goods from z_b^1 to z_b^2 can be written as

$$R_1(z_a, z_b^1, z_b^2, g) \equiv P(z_a, z_b^1, z_b^2, g)/P(z_a, 0, z_b^2, g)$$

$$= [S(z_a, z_b^1, g) - S(z_a, z_b^2, g)]/[S(z_a, 0, g) - S(z_a, z_b^2, g)]. \tag{4}$$

$R_1(z_a, z_b^1, z_b, g)$ in (4) measures the value of the change from z_b^1 to z_b^2 as a proportion of the total value of z_b^2 given in (2'). Finally, note that, in situations where (z_a, z_b^2) is on the upper bound of the feasible set, then $S(z_a, z_b^2, g) = 0$ and equation (4) reduces to

$$R_1(z_a, z_b^1, z_b^2, g) = S(z_a, z_b^1, g)/S(z_a, 0, g), \tag{4'}$$

showing that a ratio of shortage functions provides a simple relative measure of environmental changes.

3. The Value of Biodiversity

Equations (2) and (3) provide measures of the productive value of an ecosystem. However, it is often of interest to know more about the source of this value. The concerns about biodiversity provide a good example. Indeed, biodiversity issues typically arise when it is believed that the value of an ecosystem is greater than the value of its parts. This suggests the need to evaluate the value of environmental goods both for their "total value" and for the "sum of their parts." To address this issue, let I_b denote the set of environmental goods in z_b, and consider a partition of the set $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$, with $2 \le K \le n$. Let $z_{bk} = \{z_j: i \in I_{bk}\}$ denote the environmental goods in the subset I_{bk}, $k = 1, \ldots, K$, with $z_b = (z_{b1}, \ldots, z_{bK})$.

To address diversity issues, for a given $z \equiv (z_a, z_b) \in Z$, consider K situations where $z^k \equiv (z_a^k, z_b^k) \neq 0$ for $k = 1, \ldots, K$, and where $\sum_{k=1}^{K} z^k = z$. Using the shortage function (1), we propose the following measure of <u>diversity</u>

$$D(z, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g), \tag{5}$$

where $z = \sum_{k=1}^{K} z^k$. Equation (5) compares two situations involving netputs z: one where the netputs z are involved in a single production process; and the other situation where there are K <u>separate</u> production processes, with z_k being the netputs used in the k-th production process. With $z = \sum_{k=1}^{K} z^k$, it follows that, in each situation, the same aggregate amounts of resources are used to produce the same aggregate netputs. In this context, equation (5) provides a measure of the number of units of the reference bundle g that can be saved by producing z jointly (compared to producing the same aggregate netputs z in K separate production processes). Intuitively, $D(z, g) > 0$ if there are productivity gains associated with a joint use of the netputs z. This reflects that $D(z, g) > 0$ corresponds to situations where "the whole is worth more than the sum of the parts." From (5), this would be associated with the <u>subadditivity</u> of the shortage function.

To help further motivate (5), consider the case where $p\, g = 1$. Then, use equation (2) to define $P_k \equiv S(z_a, 0, g)/K - S(z^k, g)$ as measuring the value of the environmental goods in z^k, $k = 1, ..., K$, where $\sum_{k=1}^{K} z_a^k = z_a$. Note that $S(z_a, 0, g)$ is divided by K to reflect the fact that the original ecosystem is being evaluated in the context of K separate systems. Then, the value of the "sum of the parts" across the K systems is

$$\begin{aligned} \sum_{k=1}^{K} P_k &= S(z_a, 0, g) - \sum_{k=1}^{K} S(z^k, g), \\ &= P(z_a, 0, z_b, g) - D(z, g), \end{aligned}$$

using (2') and (5). It follows that $D(z, g) = P(z_a, 0, z_b, g) - \sum_{k=1}^{K} P_k$. This shows that the value of diversity $D(z, g)$ in (5) is indeed the difference between the total value of the environmental goods z_b, $P(z_a, 0, z_b, g)$, and the value of the "sum of its parts", $\sum_{k=1}^{K} P_k$.

As indicated in proposition 1, when the reference bundle g is chosen such that p g = 1, then D(z, g) in (5) provides a monetary measure of the value of diversity. As such, equation (5) provides an absolute measure of diversity. However, it can be easily used to obtain a relative measure. In situations where the total value $P(z_a, 0, z_b, g)$ in (2') is non-zero, a relative measure of diversity can be written as

$$R_D(z, g) \equiv D(z, g)/P(z_a, 0, z_b, g)$$

$$= [\sum_{k=1}^{K} S(z^k, g) - S(z, g)]/[S(z_a, 0, g) - S(z_a, z_b, g)]. \tag{6}$$

where $z \equiv (z_a, z_b) = \sum_{k=1}^{K} z^k$. $R_D(z, g)$ in (6) measures the value of diversity as a proportion of the total value of z_b given in (2'). In situations where $z \equiv (z_a, z_b)$ is on the upper bound of the feasible set, then $S(z_a, z_b, g) = 0$ and equation (6) reduces to

$$R_D(z, g) = \sum_{k=1}^{K} S(z^k, g)/S(z_a, 0, g), \tag{6'}$$

showing that a ratio of shortage functions provides a simple relative measure of diversity.

Note that equation (5) defines diversity in the general context of the netputs z, which include both the private goods z_a and the environmental goods z_b. Given our interest on biodiversity, we want to focus our attention on diversity issues related only to environmental goods. In this context, it will be useful to define $z^k \equiv (z_a^k, z_b^k)$ in (5) in a more specific way. Consider choosing

$$z_a^k = z_a/K, \tag{7a}$$

and

$$z_i^k = z_i^+ \equiv \beta_k z_i \text{ if } i \in I_{bk}, \tag{7b}$$

$$= z_i^- \equiv z_i (1-\beta_k)/(K-1) \text{ if } i \in I_b \backslash I_{bk}, \tag{7c}$$

for some $\beta_k \in (1/K, 1]$, k = 1, …, K.[2] First, note that equations (7a)-(7c) always satisfy $z = \sum_{k=1}^{K} z^k$. This guarantees that the same aggregate netputs are involved in both situations. Second, equation (7a) divides the market goods z_a equally among the K production processes. This imposes "no diversity" in the use of the private goods z_a across the K production processes. Third, equations (7b)-(7c) establish the patterns of specialization for the environmental goods z_b. The parameter β_k in b) represents the proportion of the original environmental netputs $\{z_i: i \in I_{bk}\}$ that are produced in the k-th process. And from (7c), $(1-\beta_k)/(K-1)$ represents the proportion of the original netputs $\{z_i: i \in I_b \backslash I_{bk}\}$ produced in the k-th process. When $\beta_k = 1$, this corresponds to the case of complete specialization where the k-th process relies exclusively on environmental netputs in the subset I_{bk} (with $z_{ik} = z_i$ if $i \in I_{bk}$) with $z_{ik} = 0$ for $i \in I_b \backslash I_{bk}$. If it applies for all k, each of the K process is associated with a complete loss of biodiversity in environmental goods z_b across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. Alternatively, when $\beta_k \in (1/K, 1)$, this allows for partial specialization. Then, the k-th process is associated with a partial loss of biodiversity in environmental goods z_b. Thus, the parameter $\beta_k \in (1/K, 1]$ allows for varying amount of specialization in the environmental netputs in the k-th process. Alternatively stated, the β_k's allow for varying amount of loss of biodiversity among the K processes. In general, the degree of specialization in the k-th process increases with β_k. This means that the loss in biodiversity in the K processes also increases with the β_k's.

With $z^k \equiv (z_a^k, z_b^k)$ given in (7a)-(7c), equation (5) becomes

$$D(z, \beta, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g), \tag{8}$$

where $\beta = (\beta_1, \ldots, \beta_K)$. Equation (8) provides a measure of the value of biodiversity. It measures the number of units of the reference bundle g that can be saved when the environmental goods z_b are part of a joint production process in the ecological system (compared to the case where the environmental goods z_b are part of K specialized production processes satisfying (7a)-(7c) and producing the same aggregate netputs z). Using arguments similar to the ones presented in Proposition 1 yields the following result.

Proposition 2: When the reference bundle g is chosen to have unit value (with p g = 1), then $D(z, \beta, g)$ in (8) is a monetary measure of the value of biodiversity.

4. A Decomposition

While equation (8) provides a basis to evaluate the value of biodiversity, it is of interest to identify the sources of this value. In this section, we develop a general decomposition of the benefits associated with biodiversity, thus providing new insights into its sources.

Without a loss of generality, let $S(z, g) \equiv S_v(z, g) + S_f(z, g)$. This decomposes the shortage function $S(z, g)$ into a "variable function" $S_v(z, g)$ and a "fixed function" $S_f(z, g)$. We assume that the variable shortage function $S_v(z, g)$ is continuous in z. The fixed shortage function $S_f(z, g)$ is defined as a step function with possible discontinuities around $z = 0$. The step function $S_f(z, g)$ satisfies $S_f(0, g) = 0$, and it is constant with respect to z as long as the set of non-zero netputs does not change. Thus, $S_f(z, g)$ (an hence $S(z, g)$) can exhibit jump-discontinuities in z when any netput z_i changes between zero and an arbitrarily small non-zero number. As further discussed below, the jump-discontinuities reflect the presence of catalysts (or repressors) when the presence of a small quantity of z_i generates a large increase (decrease) in ecosystem productivity. Thus, the fixed function $S_f(z, g)$ can capture "catalytic effects" when a small increase in some netputs from 0 has a large effect on productivity.

We start from the partition $I_b = \{I_{b1}, \ldots, I_{bK}\}$, where I_{bk} denotes the environmental goods that the k-th ecological process specializes in, $k = 1, \ldots, K$, with $2 \leq K \leq n$. We use the following notation. Let $z_a = \{z_i: i \in I_a\}$, $z_{bk} = \{z_i: i \in I_{bk}\}$, $z_b = (z_{b1}, \ldots, z_{bK})$, $z_{b\backslash bk} = (z_{b1}, \ldots, z_{b,k-1}, z_{b,k+1}, \ldots, z_{bK})$, and $z_{b,i:j} = (z_{bi}, z_{b,i+1}, \ldots, z_{b,j-1}, z_{bj})$ for $i < j$. From equations (7), it follows that $z^k = (z_a/K, \beta\, z_{bk}, (1\text{-}\beta)\, z_{b\backslash bk})$. Our main result is stated next. (See the proof in the Appendix).

<u>Proposition 3</u>: Given $S(z, g) \equiv S_v(z, g) + S_f(z, g)$, assume that $S_v(z, g)$ is continuously differentiable in z_b almost everywhere. Under equations (7), the value of biodiversity $D(z, \beta, g)$ in (8) evaluated at netputs $z = (z_a, z_b)$ can be decomposed as follows

$$D \equiv D_C + D_R + D_V + D_f, \tag{9}$$

where

$$D_C \equiv \sum_{k=1}^{K-1} \{ \int_{z_{bk}^-}^{z_{bk}^+} \frac{\partial S_v}{\partial \gamma} (z_a/K, z^-_{b,1:k-1}, \gamma, z^-_{b,k+1:K}, g)\, d\gamma$$

$$- \int_{z_{bk}^-}^{z_{bk}^+} \frac{\partial S_v}{\partial \gamma} (z_a/K, z^-_{b,1:k-1}, \gamma, z^+_{b,k+1:K}, g)\, d\gamma \}, \tag{10a}$$

$$D_R \equiv K\, S(z/K, g) - S(z, g), \tag{10b}$$

$$D_V \equiv S(z_a/K, z_b^+, g) + (K-1)\, S(z_a/K, z_b^-, g) - K\, S(z/K, g), \tag{10c}$$

and

$$D_f \equiv \sum_{k=1}^{K} S_f(z_a/K, z^+_{bk}, z^-_{b\backslash bk}, g) - S_f(z_a/K, z_b^+, g) - (K-1)\, S_f(z_a/K, z_b^-, g), \tag{10d}$$

Proposition 3 gives a decomposition of the value of biodiversity D(z, g) in (8) into four additive terms: D_C given in (10a), D_R given in (10b), D_V given in (10c), and D_f given in (10d).

The term D_C in (10a) depends on how $z_{b\backslash bk}$ affects the marginal shortage of z_{bk}, k = 1, …, K. It reflects the presence of <u>complementarity</u> among environmental netputs in z_b. To see that, consider the case where the shortage function is twice continuously differentiable in z_b. Then, equation (10a) can be written as

$$D_C \equiv -\sum_{k=1}^{K-1} \int_{z^-_{b,k+1:K}}^{z^+_{b,k+1:K}} \int_{z_{bk}^-}^{z_{bk}^+} \frac{\partial^2 S_v}{\partial \gamma_1 \partial \gamma_2} (z_a/K, z^-_{b,1:k-1}, \gamma_1, \gamma_2, g)\, d\gamma_1\, d\gamma_2. \tag{10a'}$$

Equation (10a') makes it clear that the sign of D_C depends on the sign of $\partial^2 S/\partial z_{bk} \partial z_{b\backslash bk}$, k = 1, …, K. As discussed above, the marginal shortage can be interpreted as the negative of the marginal product. In this context, define <u>complementarity between z_{bk} and $z_{b\backslash bk}$</u> as any situation where the shortage function satisfies $\partial^2 S/\partial z_{bk} \partial z_{b\backslash bk} < 0$. Indeed, with $\partial S/\partial z_{bk}$ reflecting the negative of the marginal product of z_{bk}, complementarity (with $\partial^2 S/\partial z_{bk} \partial z_{b\backslash bk} < 0$) means that z_{bk} has positive effects on the marginal product of $z_{b\backslash bk}$, implying positive synergies between z_{bk} and $z_{b\backslash bk}$. Then, it is clear from (10a) that $D_C > 0$ if the shortage function exhibits complementarity between z_{bk} and $z_{b\backslash bk}$, k = 1, …, K.

Thus, proposition 3 establishes that complementarity among environmental netputs (as reflected by the term D_C) is one of the components of the value of biodiversity. This supports the arguments that complementarity is an important contributing factor to the value of biodiversity (e.g., Faith et al.; Justus and Sarkar; Loreau and Hector).

To interpret the term D_R in (10b), we make use of lemma 1 in the Appendix. Given $K \geq 2$, lemma 1 implies that $K\, S(z/K, g) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} S(z, g)$ under $\begin{Bmatrix} \text{decreasing returns to scale (DRTS)} \\ \text{constant returns to scale (CRTS)} \\ \text{increasing returns to scale (IRTS)} \end{Bmatrix}$. It follows that

$$D_R \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}. \tag{10b'}$$

Equation (10b') implies that D_R vanishes under CRTS, but is positive (negative) under IRTS (DRTS). Thus, the term D_R can be interpreted as capturing <u>scale effects</u> generated as the netput vector z is produced in more specialized ways. Also, equation (10b') shows that $D_R \geq 0$ under non-decreasing returns to scale. Intuitively, more specialized processes involve smaller scales of operation. Under IRTS, such processes (associated with lower biodiversity) would appear less productive (their scale of operation is "too small"), implying that the scale effect contributes positively to the value of biodiversity ($D_R > 0$). Alternatively, under DRTS, the specialized processes would appear more productive (as the scale of operation of the integrated process is "too large"), implying a negative scale effect ($D_R < 0$).

Thus, proposition 3 establishes how the scale of ecosystem and the nature of returns to scale for the underlying process (as reflected by the term D_R) can affect the value of biodiversity. This supports the arguments that scale effects can play an important role in the evaluation of ecological functioning (e.g., Debinski and Holt; Bisonnette and Storch).

The term D_V in (10c) reflects the <u>effect of convexity</u>. To show it, we make use of lemma 2 in the Appendix. Lemma 2 states that the shortage function S(z, g) is convex in z when the feasible set Z is convex. It follows that, under the convexity of Z, the shortage function satisfies $\sum_{j=1}^{K} \theta_j\, S(z^j, g) \geq S(\sum_{j=1}^{K}$

$\theta_j z^j$, g) for $\theta_j \in [0, 1]$ satisfying $\sum_{j=1}^{K} \theta_j = 1$. Choosing $\theta_j = 1/K$, $z^1 = (z_a/K, z_b^+, g)$ and $z^j = (z_a/K, z_b^-, g)$ for j = 2, …, K, it follows from (10c) that $D_V \geq 0$. Thus, a convex technology is sufficient to imply that $D_V \geq 0$. Intuitively, a convex technology means diminishing marginal productivity, a standard characterization of resource scarcity. This suggests that the term D_V reflects the role of resource scarcity. In this context, proposition 3 shows that resource scarcity contributes positively to the value of biodiversity. Alternatively, our analysis indicates that $D_V < 0$ can arise only under a non-convex technology. The identification of such effect seems to be new in the literature. Its empirical relevance will be evaluated below.

Finally, the term D_f in (10d) reflects catalytic effects associated with discontinuous productivity effects. Indeed, in the absence of discontinuity of the shortage function S(z, g) around z = 0, then $S_f(z, g) = 0$ and thus $D_f = 0$ in equation (10d). When S_f can be non-zero, note that D_f can be positive, zero, or negative. Interestingly, we defined the fixed shortage function $S_f(z, g)$ to reflect possible discontinuities but only around z = 0. This means that $S_f(z, g)$ is a constant as along as the set of non-zero netputs does not change. Then, from equation (10d), $\beta \in (1/K, 1)$ implies $D_f = 0$. Alternatively, the catalytic component D_f can be non-zero only when some $\beta_k = 1$. It means that the catalytic effects are relevant in the value of biodiversity only when some $\beta_k = 1$, i.e., only under a complete loss of diversity in some environmental goods. In the context where $\beta_k = 1$ for all k, from equation (10d), D_f is positive if and only if $\sum_{k=1}^{K} S_f(z_a/K, z_{bk}, 0, g) > S_f(z_a/K, z_b, g) + (K-1)\, S_f(z_a/K, 0, g)$. Then, catalytic effects contribute to the value of biodiversity. This corresponds to situations where a complete loss of biodiversity generates a discontinuous and large decrease in the productivity of the specialized processes. Thus, in the presence of discontinuities related to catalytic netputs around $z_b = 0$, Proposition 3 shows how a complete loss of biodiversity can contribute positively to its value through D_f.

Proposition 3 provides useful information on conditions contributing to the value of biodiversity. It generates the following result.

Corollary 1: Sufficient conditions for a positive value of biodiversity are:

1) there is complementarity between z_{bk} and $z_{b\backslash bk}$, k = 1, …, K, ($D_C > 0$),
2) the technology exhibits non-decreasing returns to scale ($D_R \geq 0$),
3) the technology Z is convex (with $D_V \geq 0$), and
4) $D_f \geq 0$.

Thus, the value of biodiversity can arise from complementarity among environmental goods in z_b ($D_C > 0$), from increasing returns to scale ($D_R > 0$), from a convex technology ($D_V \geq 0$), and/or from catalytic effects (when $D_f \geq 0$). This identifies the role of complementarity as an important contributing factor to the value of biodiversity. However, it also shows that complementarity is in general neither necessary nor sufficient to generate a positive value for biodiversity. For example, under decreasing returns to scale (DRTS), equation (10b') implies that $D_R < 0$. This reflects the fact that, under DRTS, the smaller and more specialized processes require fewer resources to produce the same aggregate outputs. When this scale effect dominates the other components in (9), then D < 0, i.e. biodiversity would have a negative value even in the presence of complementarity. Alternatively, B_V can become negative under a non-convex technology. Again if this negative convexity effect dominates the other components in (9), then D < 0, and biodiversity would have a negative value even in the presence of complementarity. Finally, the catalytic effect D_f is present only under complete loss of biodiversity in environmental goods across at least some elements of the partition $I_b = \{I_{b1}, I_{b2}, …, I_{bK}\}$. Scenarios where D_f is positive and large can arise when catalytic netputs associate a complete loss of biodiversity with a large decline in the productivity of the specialized processes. This illustrates the usefulness of the decomposition provided in Proposition 3. However, the relative importance of each component is likely to be specific to each ecosystem. This stresses the need for empirical analyses.

5. Empirical application

This section presents an empirical illustration of the methodology discussed above. Our empirical analysis focuses on an agroecosystem where inputs $x = (x_1, x_2, \ldots)$ are being used to produce outputs $y = (y_1, y_2, y_3, \ldots)$, with $z \equiv (y, -x) \in Z$. Choosing $g = (1, 0, \ldots, 0)$ and assuming that y_1 is a marketed output satisfying free disposal, it follows that the production technology Z can be written as $Z = \{(y, -x): y_1 - S(y_2, y_3,\ldots, -x, g) \leq 0, (y, -x) \in R^{m+n}\}$, where $S(y_2, y_3,\ldots, -x, g)$ is the shortage function defined in (1). Letting $F(y_2, y_3,\ldots, x) \equiv S(y_2, y_3,\ldots, -x, g)$, the frontier technology is represented by the multi-output production function $y_1 = F(y_2, y_3,\ldots, x)$. An empirical application of our methodology then requires the specification and estimation of the production function $y_1 = F(y_2, y_3,\ldots, x)$.

First, we specify a parametric form for the production function: $F(y_2, y_3,\ldots, x) = f(y_2, y_3,\ldots, x, \beta)$, where β is a set of parameters to estimate. Second, we add an error term to generate the econometric model

$$y_1 = f(y_2, y_3,\ldots, x, \beta) + e, \tag{11}$$

where e is a random variable distributed with mean zero and finite variance. Equation (11) is an econometric model that can be used to generate a consistent estimate β^e of β. This gives a consistent estimate of the mean shortage function: $E[S(y_2, y_3,\ldots, -x, g)] = f(y_2, y_3,\ldots, x, \beta^e)$. Using equations (5), (9) and (10), such estimate provides a basis to investigate empirically the value of biodiversity.

The estimation of equation (11) poses at least two econometric challenges. First, we would like $f(y_2, y_3,\ldots, x, \beta)$ to provide a flexible representation of the effects of outputs $(y_2, y_3,\ldots)$ on the productivity of the ecosystem. This is feasible when the number of outputs remains small. However, this becomes problematic if the number of outputs becomes large (e.g., more than 5). Indeed, a flexible representation of output effects with a large number of outputs requires a large number of parameters, implying the prospects of facing severe collinearity problems. In other words, the econometric estimation of (11) can become problematic if the number of outputs is large. Second, when applied to an agroecosystem, equation (11) involves netputs that are subject to direct management. This means that the choice of (y, x) generates the possibility of endogeneity issues. Indeed, if the netput decisions for (y, x) depend on information that is not available to econometrician, then they may become correlated to the error term e in (11), implying the presence of endogeneity bias. This bias implies that standard estimation methods (e.g., least squares) will provide biased and inconsistent parameter estimates. This suggests the need to address endogeneity issues explicitly by using appropriate estimation methods. This can be done by using instrumental variable estimation methods that provide consistent parameter estimate in the presence of endogeneity.

Our empirical analysis focuses on a situation with three outputs. Three is "large enough" to allow the investigation of the benefit of diversity across agroecological processes, yet "small enough" to avoid collinearity problems. In this context, with three outputs, we specify (11) to be quadratic function of outputs y. This provides a parsimonious specification allowing for a flexible representation of how each output affects the marginal product of other outputs. We also assume that inputs x enter (11) in log form.[3] To address endogeneity issues, we adopt an instrumental variables estimation approach.

6. Site description and data

The dataset used in the analysis is from a farm survey conducted in 1999 and 2000 in the highlands (more than 1500 meters above sea level (masl)) of Tigray region of Ethiopia by researchers from Mekelle University, the International Food Policy Research Institute (IFPRI), and the International Livestock Research Institute (ILRI). The survey involved a stratified sampling of farm households, with the strata being chosen according to agricultural potential, market access, and population density (Pender et al. 2001). In the Tigray region, peasant associations (PAs) were stratified by distance to the *woreda* town (greater or less than 10 km). Three strata were defined, with 54 PAs randomly selected across the strata. PAs closer to towns were selected with a higher sampling fraction to assure adequate representation. From each of the remaining PAs, two villages were randomly selected, and from each village, five households were randomly selected. A total of 50 PAs, 100 villages, and 500 households were then surveyed. Usable data were available for 96 villages, or kushets. After controlling for outliers and observations with missing values for relevant variables, 292 household observations remained. We screened the production of these households and identified teff, barley and wheat as the most widely grown crops. Cereal production is the most relevant agricultural production in the studied area. Ethiopia is a biodiversity "hotspot." It is, indeed, a recognized global center of genetic diversity for several cereal crops (Vavilov; Harlan). Farms are diversified thus different plots are allocated to different cereal crops. Thus our analysis of agro-biodiversity focuses on three outputs: teff, barley and wheat. The use of conventional inputs is minimal. Farmers rely mostly on labor and oxen power. Agroecological conditions can be challenging because of pervasive land degradation and erratic rainfall. This setting, therefore, provides a prime example to empirically investigate the contribution to production of biodiversity. Biodiversity's gains are indeed expected to be high when agroecological conditions are more difficult (Callaway and Walker).

7. Econometric Results

Using farm-level data from the Ethiopian survey, equation (11) was specified and estimated by instrumental variable method. The analysis covers three outputs: y_1 = teff, y_2 = barley and y_3 = wheat. The inputs x include animal traction, land, labor, fertilizer and rainfall. A number of additional variables were added to capture the heterogeneity in agro-climatic conditions across observations in the sample. They include soil fertility, soil erosion, slope, location, improved seeds, and the presence of soil conservation practices. Equation (11) was specified to be quadratic in outputs (y_2, y_3), linear in the logarithm of land, labor and animal traction, and linear in other variables. The quadratic output terms allow for flexible patterns of marginal productivity, including the effect of an output on the marginal product of other outputs.

To address the issue of endogeneity, we rely on instrumental variable estimation. We identified a set of suitable instruments following both theory and existing literature (Pender et al.). In the absence of serial correlation in the error term, lagged variables for output, farm agroecological heterogeneity, land share under conservation measure and distance from input supplier are suitable instruments. To assess the validity of these instruments, both Hausman type and regression residuals testing procedures for endogeneity were applied. We found that outputs variables (barley and wheat), their interaction and soil conservation measure to be endogenous. The Sargan-Hansen test using the over-identifying restrictions was used to investigate whether the orthogonality conditions between the instruments and the error term are satisfied. We failed to reject the null hypothesis. Therefore the choice of instruments seems appropriate. The results of the tests are provided at the bottom of table 3. The null hypothesis of homoscedasticity was tested against the alternative hypotheses of *i)* general heteroscedasticity, and *ii)* multiplicative heteroscedasticity. Tests results confirmed the presence of heteroscedasticity and that multiplicative heteroscedasticity was present. To obtain efficiency gains, we therefore implemented a weighted estimation method using weights obtained from the consistent estimate of the error variance.

We also investigated whether the production function $f(\cdot)$ in (11) exhibited discontinuities at $y = 0$. This was done by introducing dummy variables equal to 1 if $y_i = 0$ and zero otherwise, and testing their statistical significance. Using a Wald test and a 10 percent significance level, we failed to reject the null hypothesis that these dummy variables have a significant effect on productivity. Thus, we did not find statistical evidence that the production function $f(\cdot)$ was discontinuous at $y = 0$. This means that we did not find statistical evidence of significant "catalytic effects". On that basis, our analysis proceeds assuming that the production function $f(\cdot)$ in (11) is continuous everywhere.

Table 3 reports the estimation results. The conventional inputs (animal traction, land, labor and fertilizer) are all positive and statistically significant. The estimated coefficients for barley, wheat and wheat show statistical significance. The coefficient of the linear term for barley is negative and statistically significant at the 5 percent level. And the interaction term (barley × wheat) is positive and statistically significant at the 5 percent level. This indicates the presence of positive interaction effects across crops. Such positive interaction effects on productivity give a hint about the presence of complementarity in the agro-ecosystem. Such effects and their implications for the value of diversity are further evaluated below. In Table 3, while the coefficients related to agroecological conditions are consistent with expectation, none of them is statistically significant at the 10 percent level. A negative and strongly significant coefficient was estimated for "East". This is consistent with evidence that the eastern part of the region has the worst conditions for agricultural production (Gebremedhin et al.).

The use of improved seeds seems not relevant. Indeed, the estimated coefficient is not statistically significant. The share of land under reduced tillage is found to have an important impact on productivity. The estimated coefficient, indeed, is positive and statistically significant. This result indicates that soil conservation measures can be a win-win strategy in such agricultural system.

8. Implications

The estimated production function (reported in Table 3) provides a basis for investigating the productivity of the agroecosystem. Of special interests are the implications for the value of diversity D given in (8) and its components given in (9) and (10): scale effect D_R, complementarity effect D_C, and convexity effect D_V.[4] In this context, based on the estimated production function, bootstrapping is used to simulate the distribution of D and its components. This provides a basis for assessing both the magnitude of the diversity measures and their statistical significance. The simulation results are presented in Tables 4, 5 and 6.

Table 4 shows the diversity measure D and its components: complementarity D_C, scale D_R and convexity D_V, evaluated for a farm being 1.3 times the sample mean and facing a degree of specialization β equal to 0.8.[5] Table 4 shows that both convexity effect and complementarity effect are statistically significant. The complementarity effect D_C is found to be positive and significant at the 5 percent level. This provides evidence that each crop tends to have a positive effect on the marginal productivity of other crops. Comparing D_C = 14.257 with an average teff production of 151, the productivity benefit associated with complementarity amounts to a 9.3 percent boost in productivity. This documents that the complementarity component of diversity provides significant productivity benefits to the functioning of the agro ecosystem.

The scale component D_R in Table 4 is positive (2.20) but not statistically significant. As shown in equation (10b'), $D_R = 0$ under constant returns to scale (CRTS). This indicates that the scenario evaluated in Table 4 corresponds to a situation where CRTS cannot be rejected. We also conducted the analysis reported in Table 4 under different farm sizes. We did find some evidence that D_R became positive and statistically significant for very small farm sizes. This indicates the presence of increasing returns to scale (IRTS) for very small farm sizes, where $D_R > 0$ under IRTS means that scale effects can contribute positively to the value of diversity. However, such evaluations involved simulating farm sizes that were at the lower bound of the ones observed in the sample. This means that the statistical evidence in favor of IRTS has to be interpreted with caution: it is always dangerous to try to extrapolate outside of the sample information. We found that, within the range of most farm sizes observed in the sample, D_R was not statistically different from zero (as in the case reported in Table 4). This means that, for most farms, the technology of the agro-ecosystem seems to exhibit CRTS (in which case $D_R = 0$), except possibly for very small farms (where IRTS may arise, with $D_R > 0$).[6] In other words, for most farms, the evidence against CRTS is weak, implying that the scale effect D_R does not appear to be an important part of the value of diversity in our agroecosystem.

Table 4 shows that the convexity effect D_V is negative: -12.59. And it is statistically significant at the 1 percent level. As discussed above, D_V is expected to be positive under convex technology (i.e. a technology exhibiting decreasing marginal returns). This provide evidence that our agro-ecosystem does not exhibit decreasing marginal returns, and that its underlying technology is <u>not convex</u>. Moreover, this non-convexity means that the convexity component D_V provides an incentive to specialize. Comparing $D_V = -12.59$ with an average teff production of 151, the productivity loss associated with (non)convexity amounts to a 8.33 percent decline in productivity. Besides being statistically significant, this also appears to be economically important. In other words, our empirical analysis indicates that non-convexity in the technology of the agro-ecosystem provides disincentives to diversify and contributes to reducing the value of diversity.

When putting all components together, Table 4 shows that the value of diversity D remains positive: D = 3.97. This amounts to a 2.6 percent contribution to productivity. This reflects the fact that the complementarity component (D_C = 14.24) is large enough to dominate the negative convexity component (D_V = - 12.59). Even if we ignore the (non-significant) scale component D_R, note that $D_C + D_V$ = 1.663 is positive and contributes to a 1.1 percent boost in productivity. However, neither D nor (D_C + D_V) is statistically different from zero at the 5 percent significance level. This means that the evidence of significant overall value of diversity in our agro-ecosystem is weak. The reason is that, even in the presence of significant complementarity benefits, such benefits are cancelled out by opposite effects from the (non) convexity component.

Table 5 presents simulation results evaluating the effects of the degree of specialization β on the complementarity component D_C and the convexity component D_V. It shows that both D_C and D_V are small under mild specialization (e.g., β = 0.4). However, their magnitude increases rapidly with β. For example, under complete specialization (β = 1), D_C rises to 29.10 while D_V becomes –25.70. This amounts to a 19.2 percent increase and a 16.6 fall in productivity, respectively. These magnitudes indicate large and significant impacts of each component on agro system productivity. In all cases, the magnitude of the complementarity component dominates the magnitude of the (non) convexity component. This means that their combined effect ($D_C + D_V$) is always positive. However, these two effects tend to cancel each other, implying that their combined effect tends to be small and is no longer statistically significant. Thus, the evidence of non-significant overall value of diversity reported in Table 4 remains valid under alternative diversification schemes.

Table 6 presents simulation results evaluating the effects of farm size on the complementarity component D_C and the convexity component D_V. It shows that, although they remain statistically significant, both D_C and D_V tend to be small on small farms. However, their magnitude increases rapidly with farm size. For example, for farm size equaled to 1.5 times the sample mean, D_C rises to 38.74 while D_V becomes -34.22. These magnitudes indicate large and significant impacts of each component on larger farms. This provides evidence that, in absolute value, both the complementarity component and the (non)convexity component increase with farm size. In all cases, the magnitude of the complementarity component dominates the magnitude of the (non)convexity component. This means that their combined effect ($D_C + D_V$) is always positive. Again, these two effects tend to cancel each other, implying that their combined effect is no longer statistically significant. This indicates that the evidence of non-significant overall value of diversity reported in Table 4 remains valid for a wide range of farm sizes.

9. Concluding Remarks

We have presented an analysis of the value of biodiversity in an ecosystem. First, a conceptual framework was developed to assess the productive value of biodiversity. The analysis applies under general conditions, allowing for non-convexities, lack of free disposal in environmental goods, and dynamics. We relied on Luenberger's shortage function to provide a measure of the productive value of biodiversity. When positive, this value reflects the fact that an ecosystem is worth more than the "sum of its parts." We showed that this value can be decomposed into four additive components, reflecting complementarity effects, scale effects, convexity effects, and catalytic effects. This provides new and useful information on its sources and determinants.

Second, the usefulness of the approach was illustrated with an application to crop biodiversity of an agroecosystem in Ethiopia. This helps inform the on-going debate on the value of biodiversity in agroecosystem. The empirical analysis involved the specification and estimation of the shortage function as a representation of the underlying technology. Relying on an instrumental variables estimator, the estimates were used to evaluate the productive value of biodiversity and its components. Results show that the value of crop diversity is positive.

The complementarity effect was found to positive and significant at the 5 percent level. In the context of the Ethiopian agroecosystem, this provides evidence that each crop tends to stimulate the marginal productivity of other crops. The complementarity effect is estimated to generate a 9.3 percent increase in productivity. Thus, our analysis shows that complementarity provides a positive and significant contribution to the productive value of crop diversity in the Ethiopian agroecosystem.

We also found evidence that the convexity component of diversity value is negative and statistically significant. This corresponds to a technology that is not convex, i.e. where marginal products of outputs are not diminishing. This means that the convexity component provides an incentive to specialize. In general, the (negative) convexity component is dominated by the (positive) complementarity component, generating a positive overall value of diversity. However, as these two terms tend to cancel each other, our estimate of the overall value of diversity is not statistically significant. This indicates that, in the Ethiopian agroecosystem, the complementarity effects are not large enough to provide a strong productivity incentive to diversify.

Finally, our empirical analysis did not find statistical evidence that either the scale effect or the catalytic effect played a significant role in the value of biodiversity. The lack of evidence of a scale effect means that farm size does not have a large impact on the functioning of the agroecosystem in Ethiopia. However, our empirical results did suggest that both complementarity effects and convexity effects may increase with farm size.

While our investigation focused on the productive value of biodiversity, we should note that this value is only a part of the total value of an ecosystem. This indicates the need to place the analysis in the broader context of ecological-economic interactions. This would include the value of biodiversity to consumers. Under uncertainty, this means examining the role of risk preferences and their implications for the design and implementation of risk management schemes. And in a dynamic context, this would include addressing the issue of how new information that becomes available over time is used in ecosystem management. Finally, while our analysis of an Ethiopian agroecosystem illustrated the usefulness of our approach to biodiversity valuation, we should keep in mind that our empirical findings may not apply to alternative ecosystems. There is a need for additional empirical investigations of the productivity implications of ecosystem functioning. These appear to be good topics for future research.

Appendix

Proof of Proposition 3:

Let $z_{b,i:j} = (z_{bi}, z_{b,i+1}, \ldots, z_{b,j-1}, z_{bj})$ for $i < j$. Given $S(z, g) = S_v(z, g) + S_f(z, g)$, and using (5) and (7), the value of diversity is

$$D \equiv \sum_{k=1}^{K} S(z_a/K, z_{bk}^{+}, z_{b\backslash bk}^{-}, g) - S(z, g),$$

$$= \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{+}, z_{b,k+1:K}^{-}, g) + S_v(z_a/K, z_{b,1:K-1}^{-}, z_{bK}^{+}, g)$$

$$+ \sum_{k=1}^{K} S_f(z_a/K, z_{bk}^{+}, z_{b\backslash bk}^{-}, g) - S(z, g), \qquad \text{(A1)}$$

Note that

$$\sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{+}, z_{b,k+1:K}^{+}, g) =$$

$$\sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{-}, z_{b,k+1:K}^{+}, g) + S_v(z_a/K, z_{b}^{+}, g) - S_v(z_a/K, z_{b,1:K-1}^{-}, z_{bK}^{+}, g). \qquad \text{(A2)}$$

Also,

$$\sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{-}, z_{b,k+1:K}^{-}, g) = (K-1)\, S_v(z_a/K, z_{b}^{-}, g). \qquad \text{(A3)}$$

Adding (A2) and (A3) to (A1) gives

$$D = \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{+}, z_{b,k+1:K}^{-}, g) + S_v(z_a/K, z_{b,1:K-1}^{-}, z_{bK}^{+}, g)$$

$$+ \sum_{k=1}^{K} S_f(z_a/K, z_{bk}^{+}, z_{b\backslash bk}^{-}, g) - S(z, g)$$

$$+ \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{-}, z_{b,k+1:K}^{+}, g) + S_v(z_a/K, z_{b}^{+}, g) - S_v(z_a/K, z_{b,1:K-1}^{-}, z_{bK}^{+}, g)$$

$$- \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{+}, z_{b,k+1:K}^{+}, g)$$

$$+ (K-1)\, S_v(z_a/K, z_{b}^{-}, g) - \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{-}, z_{b,k+1:K}^{-}, g).$$

This can be alternatively written as

$$D = \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{+}, z_{b,k+1:K}^{-}, g) - \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{-}, z_{b,k+1:K}^{-}, g)$$

$$- \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{+}, z_{b,k+1:K}^{+}, g) + \sum_{k=1}^{K-1} S_v(z_a/K, z_{b,1:k-1}^{-}, z_{bk}^{-}, z_{b,k+1:K}^{+}, g)$$

$$- S(z, g) + K\, S(z/K, g)$$

$$- K\, S(z/K, g) + S_v(z_a/K, z_b^+, g) + (K\text{-}1)\, S_v(z_a/K, z_b^-, g)$$

$$+ \sum_{k=1}^{K} S_f(z_a/K, z_{bk}^+, z_{b\backslash bk}^-, g). \tag{A4}$$

Assuming that $S_v(z, \cdot)$ is continuous everywhere and continuously differentiable almost everywhere on R^{m+n}, and using $S_v(z, g) = S(z, g) - S_f(z, g)$, this yields

$$S = \sum_{k=1}^{K-1} \{ \int_{z_{bk}^-}^{z_{bk}^+} \frac{\partial S_v}{\partial \gamma} (z_a/K, z_{b,1:k-1}^-, \gamma, z_{b,k+1:K}^-, g)\, d\gamma$$

$$- \int_{z_{bk}^-}^{z_{bk}^+} \frac{\partial S_v}{\partial \gamma} (z_a/K, z_{b,1:k-1}^-, \gamma, z_{b,k+1:K}^+, g)\, d\gamma\}$$

$$+ K\, S(z/K, g) - S(z, g)$$

$$+ S(z_a/K, z_b^+, g) + (K\text{-}1)\, S(z_a/K, z_b^-, g) - K\, S(z/K, g)$$

$$+ \sum_{k=1}^{K} S_f(z_a/K, z_{bk}^+, z_{b\backslash bk}^-, g) - S_f(z_a/K, z_b^+, g) - (K\text{-}1)\, S_f(z_a/K, z_b^-, g), \tag{A5}$$

which gives equations (9)-(10).

Lemma 1: For any $k \in (0, 1)$,

$$S(k\, z, g) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} k\, S(z, g) \text{ under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}.$$

Proof: By definition, the technology Z exhibits increasing returns to scale (IRTS), constant returns to scale (CRTS) or decreasing returns to scale (DRTS) if $\alpha Z \subset Z$, $\alpha Z = Z$, or $\alpha Z \supset Z$, respectively, for all $\alpha > 1$. Let $k \in (0, 1)$. Consider the case where there is a γ satisfying $(k\, z_a - \gamma\, g, k\, z_b) \in Z$. Then

$$S(k\, z, g) = \min_\gamma \{\gamma: (k\, z_a - \gamma\, g, k\, z_b) \in Z\},$$

$$= k \min_\delta \{\delta: (z_a - \delta\, g, z_b) \in (1/k)\, Z\}, \text{ where } \delta = \gamma/k,$$

$$\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} k\, S(z, g) \text{ when } (1/k)\, Z \begin{Bmatrix} \supset \\ = \\ \subset \end{Bmatrix} Z, \text{ i.e., under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}.$$

Lemma 2: If the set Z is convex, the shortage function S(z, g) is convex in z.

Proof: Consider any two netput vectors $z \in R^{n+m}$ and $z' \in R^{n+m}$. First assume that S(z, g) and S(z', g) are finite. It follows that $(z - S(z, g)\, g) \in Z$ and $(z' - S(z', g)\, g) \in Z$. Let $z'' = \theta z + (1-\theta) z'$, for any scalar $\theta \in [0, 1]$. If the set Z is convex, it follows that

$$[z'' - \theta S(z, g)\, g - (1-\theta) S(z', g)\, g] \in Z.$$

The shortage function being defined as a minimum in (1), this yields

$$S(z'', g) = S(\theta z + (1-\theta) z', g) \leq \theta S(z, g) + (1-\theta) S(z', g).$$

Second, consider the case where S(z, g) and/or S(z', g) are infinite. Then, the above inequality always holds. Thus, the function S(z, g) is convex in z.

Figure 1: An illustration of the technology

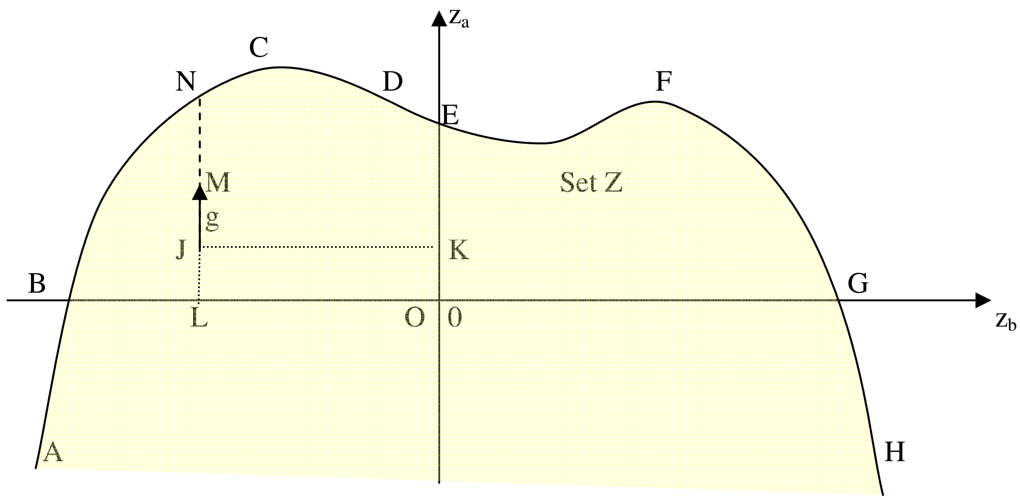

Table 1. Variable Description

Outputs	Teff	Quantity produced in Kg
	Barley	Quantity produced in Kg
	Wheat	Quantity produced in Kg
Conventional inputs	Animal traction	Animal traction in oxen days
	Land	Land for cereals in M^2
	Labor	Labor in person days
	Fertilizer	Fertilizer use in Kg
Agroecological Conditions	Rainfall	Rainfall in mm/year
	Soil fertility	Share of land classified as high fertility
	Soil Erosion	Share of land affected by severe erosion and water logging
	Slope	share of land on steep slope
Location Dummies	South	Location dummy
	East	Location dummy
	West	Location dummy
Adoptions	Improved seeds	Adoption of improved seeds (Yes=1; No=0)
	Soil conservation	Share of land under reduced tillage

Table 2. Descriptive Statistics

	Mean	Standard deviation	Minimum	Maximum
Teff	151.007	207.685	0	1292
Barley	179.521	235.828	0	1363
Wheat	82.1179	142.365	0	777
Animal traction	28.825	19.8131	2	144
Land	6631.93	4694.81	612	43194
Labor	86.2286	53.1831	15	429
Fertilizer	18.8431	23.1647	0	150
Rainfall	648.909	120.912	420.4	893.55
Soil fertility	0.092857	0.290752	0	1
Soil Erosion	0.442857	0.497613	0	1
Slope	0.088525	0.219294	0	1
South	0.264286	0.441742	0	1
East	0.275	0.447314	0	1
West	0.128571	0.335324	0	1
Improved seeds	0.107143	0.309849	0	1
Soil conservation	0.10861	0.263086	0	1

Table 3. Shortage function estimation (IV estimates)

Variables	Coefficients	Standard Errors
Constant	-291.94***	71.75
Barley	-0.461**	0.2
Barley^2	0.0002	0.0002
Wheat	-0.46	0.3
Wheat^2	-0.000137	0.00023
Barley*wheat	0.00117**	0.0006
Animal traction	67.8***	17.98
Land	19.64**	8.99
Labor	84.26***	16.85
Fertilizer	0.85*	0.5
Rainfall	0.0361	0.176
Soil fertility	22.08	29.61
Soil erosion	-9.97	16.33
Slope	-49.8	32.7
South	-27.86	20.97
East	-72.62***	28.167
West	40.5[a]	29.98
Improved seeds	6.68	37.14
Soil conservation	126.08**	62.03

N= 292;
Hansen J test: Chi-square test statistic 3.56 (with 2 degrees of freedom), P-value = 0.168.
Wu-Hausman test for endogeneity: F test statistic 6.013 (with 4 and 303 degrees of freedom), P-value < 0.001.
Durbin-Wu-Hausman test: chi-square test statistic 22.94 (with 4 degrees of freedom), P-value = 0.0001.
Breush Pagan Test for heteroscedasticity: chi-square test statistic 193.92 (with 18 degrees of freedom), P-value < 0.001.
Significance levels are denoted by one asterisk (*) at the 10 percent level, two asterisks (**) at the 5 percent level, three asterisks (***) at the 1 percent level.

Table 4. Simulated Measure of Diversity D and its Decomposition (complementarity effect D_C, scale effect D_R, and convexity effect D_V)*

Diversity	$D = D_C + D_R + D_V$	D_C	D_R	D_V	$D_C + D_V$
Diversity measure	3.966	14.257	2.203	-12.594	1.663
Standard error	(80.316)	(7.937)	(77.359)	(4.136)	(6.805)
P-value for testing $D_. = 0$	0.480	0.036	0.491	0.001	0.405

* Evaluated at a farm size equal to 1.3 times the sample mean, and at a degree of specialization $\beta = 0.8$.

Table 5. Simulated Effects of the degree of specialization β on complementarity D_C and convexity D_V*

Measure of diversity	D_C		D_V		$D_C + D_V$	
Degree of specialization β	Diversity measure	Standard error	Diversity measure	Standard error	Diversity measure	Standard error
$\beta = 0.4$	0.291**	0.162	-0.257***	0.084	0.034	0.139
$\beta = 0.6$	4.655**	2.592	-4.112***	1.350	0.543	2.222
$\beta = 0.8$	14.257**	7.937	-12.594***	4.136	1.663	6.805
$\beta = 1$	29.097**	16.198	-25.703***	8.441	3.394	13.888

* Evaluated at a farm size equal to 1.3 times the sample mean.
Significance levels are denoted by one asterisk (*) at the 10 % level, two asterisks (**) at the 5 % level, three asterisks (***) at the 1 percent level.

Table 6. Simulated Effects of farm size on complementarity D_C and convexity D_V*

Measure of diversity	D_C		D_V		$D_C + D_V$	
Farm size (proportion of sample mean)	Diversity measure	Standard error	Diversity measure	Standard error	Diversity measure	Standard error
0.5	4.304**	2.396	-3.802***	1.249	0.502	2.054
0.7	8.436**	4.696	-7.452***	2.447	0.984	4.027
1	17.217**	9.584	-15.209***	4.995	2.008	8.218
1.3	29.097**	16.198	-25.702***	8.441	3.394	13.888
1.5	38.738**	21.565	-34.219***	11.238	4.518	18.490

* Evaluated at a degree of specialization $\beta = 1$.
Significance levels are denoted by one asterisk (*) at the 10 % level, two asterisks (**) at the 5 % level, three asterisks (***) at the 1 percent level.

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Footnotes

[1] Another hypothesis is the "sampling effect" stating that increasing diversity improves the chances that specific species would be adapted to particular ecological condition (Tilman et al.).

[2] This extends the analysis presented by Chavas and Kim. By allowing the β_k's to vary, our approach can capture heterogeneous patterns of specialization.

[3] Alternative specifications were also estimated. In general, they provided results that were qualitatively similar to the ones reported below.

[4] Given the lack of statistical evidence about catalytic effects, the estimated model implicitly assumes that $D_f = 0$.

[5] Alternative degrees of specialization were also explored. See below.

[6] We also conducted the analysis assuming constant returns to scale (CRTS). This was done by defining all inputs and outputs on a per-hectare basis. As shown in (10b'), this implied $D_R = 0$. The estimates of the value diversity and its components were similar to the ones reported in the paper. This is consistent with the test results reported in Table 4 showing no strong evidence against the hypothesis $D_R = 0$.