

Pooling Data in Poverty Mapping*

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Abstract

The poverty mapping method developed in Elbers et al. (2002a) is a widely used method to study the distribution of poverty and has been applied to many countries. One issue which is unresolved in this method is whether data from different strata should be pooled or not. This paper studies the possible gains and losses from pooling and uses simulation to evaluate the reliability of a few methods that can be used to make pooling decisions. I find that one of the methods I suggest in general works better than the others and can be conducted to make pooling decisions while producing poverty maps.

1 Introduction

The poverty mapping method developed in Elbers et al. (2002a), henceforth abbreviated with the ELL method, has been applied to many countries by the World Bank in its poverty mapping projects: Azerbaijan (Baschieri and Falkingham 2004), Bangladesh (Bangladesh Bureau of Statistics and the United Nations World Food Program 2004), Brazil (Elbers et al. 2001), Bulgaria (Ivaschenko 2004), Cambodia (Fujii 2003), Laos (Van der Weide 2004), Madagascar (Mistiaen and Ozler 2002), Morocco (Lanjouw 2004), Mozambique (Simler and Nhate 2003), Philippines (Haslett and Jones 2005), Thailand (Healy et al. 2003), South Africa (Alderman et al. 2001), Vietnam (Minot et al. 2003) etc. The method is also incorporated into the World Bank software program PovMap.

The method allows researchers to estimate poverty indicators at a disaggregated sub-regional level by combining a bigger sample (e.g. data from a census) and a smaller sample (e.g. data from a survey). The smaller sample is only representative at a level higher than the sub-regional level. Poverty often differs not only at the aggregated national or regional level but also at the much more disaggregated sub-regional level. As it is often too costly to collect data to estimate poverty indicators at the sub-regional level directly, the ELL method is widely adopted to produce poverty maps at the disaggregated level. The results often provide important insights for policy making. Applying the ELL method² requires a consumption model to be built based on the survey data. The estimates of the coefficients in this model are used

²Section 2 will provide more description on the ELL method.

to predict the consumption in the census using the available information provided by the census data. Thus building consumption models is an essential step in the ELL procedure. The survey sample is usually not a random sample from the census but often a stratified sample, and the survey is representative at the stratum level. The number of strata varies from country to country. For example, the Bulgaria survey used in Ivaschenko (2004) only consists of 3 strata, whereas the survey from Thailand used in Healy et al. (2003) consists of 76 strata. The regression results of consumption models and the estimates of poverty indicators can depend dramatically on whether models are built on data from one stratum only or data from several strata. Building consumption models stratum by stratum has its advantage. Tarozzi and Deaton (2007) argue that producing poverty maps using the ELL method requires a degree of spatial homogeneity. Combining data from different strata to build a single consumption model will certainly introduce more spatial heterogeneity.

Currently, most of the poverty maps are produced based on consumption models built stratum by stratum. Exceptions are projects done for Bangladesh (Bangladesh Bureau of Statistics and the United Nations World Food Program 2004), the Philippines (Haslett and Jones 2005), Morocco (Lanjouw 2004) and Vietnam (Minot et al. 2003). In the Bangladeshi cases, one model is built for all the strata and it is argued that a single model is more “stable”: “This has the advantage of more stable parameter estimates and a better chance of finding genuine relationships that apply outside of the estimation data” (Bangladesh Bureau of Statistics and the United Nations World Food Program 2004, p.14). However, the paper does not provide evidence on

how a single model is more “stable” than stratum by stratum models. It is also not very clear what “stable” means. In the Philippine case, Haslett and Jones (2005) argue that fitting models stratum by stratum may encounter the problem of overfitting. They experiment with both fitting one single model for all the strata and fitting separate models stratum by stratum. They find that the estimates from these two ways of modeling are not far away from each other.

In the Vietnamese case, the authors built two consumption models: one for urban areas and one for rural areas. Their argument for doing so is that building consumption models stratum by stratum makes the results not satisfactory: they have low fit, insignificant coefficients and the “wrong” sign of the coefficients. The arguments are not very clear. Since the aim of building consumption models is to find the variables which can best explain consumption, insignificant variables are usually not used in practice. As the consumption model only serves a purely descriptive role³, the signs of the coefficients do not matter. It is possible that with the variables that best explain consumption the fit is still low, but it is not clear that the fit of this stratum can be improved by adding observations from another stratum. In the Moroccan case (Lanjouw 2004), consumption models are built on data from 3-5 strata because the survey covers too few observations for some strata (less than 100). It is definitely a problem to build a model with too few observations, but it is not clear that combining data from several strata can solve the problem and produce better estimates. Lanjouw also warns that the estimates from these strata should be treated with caution.

³See section 2 for more details.

Besides these arguments, one of the most obvious and popular arguments in favor of pooling is that when the number of strata is large, pooling can save a lot of time in producing the maps. The most dramatic illustration of this is the Thai case: building one model for all strata is clearly less time consuming than building 76 consumption models.

Pooling data or not thus remains as an unresolved issue in applying the ELL method for poverty mapping. In this paper, I investigate this issue. Since building consumption model is the most important step in poverty mapping, it seems that the most crucial criterion to make pooling decision is whether the pooled model can pass some sort of a “pooling test”. The most often used method is the Chow test proposed in Chow (1960). It tests the equality between sets of coefficients in two linear regressions. The test is easy to do but applying it to poverty mapping encounters many problems. The first is that almost all the pooling decisions are rejected by the Chow test. This is very understandable as there are usually 20 to 30 variables in the consumption models used in poverty mapping. The second problem is that the Chow test can only tell the equality between the coefficients in the consumption models but cannot tell anything about the estimates of the poverty indicators. It is well possible that two different sets of coefficients can in the end produce similar estimates of the poverty indicators.

To overcome these two weak points of applying the Chow test to poverty mapping, I propose three methods and use simulation to check their reliability in making pooling decisions. In the first method, I use the survey data alone to do the poverty mapping exercise and instead of looking at the coefficients in the consumption model I look at the estimates of poverty in-

dicators (henceforth abbreviated with the “survey map method”). Like the Chow test, this method only uses the survey data to decide whether data can be pooled or not. I find that this method also does not work very well. The reason is probably that whether data can be pooled or not also depends on the properties of the census data, which are not taken into account in this method.

Haslett and Jones (2005) give some hints on other ways to make pooling decisions. In their paper, they try two ways of modeling and decide to use a single model to fit all the data. Their estimates from these two ways of modeling are indeed very similar (Haslett and Jones 2005, Table 5.1). Therefore, in the second method I use both the survey and the census data to do the poverty mapping exercise. I then compare the estimates of the poverty indicators from the pooled data to that of the non-pooled data (henceforth abbreviated with the “compare values method”). This method has its advantages: it looks at the estimates of the poverty indicators instead of the coefficients in the consumption model, and it takes the census into consideration since the estimates of the poverty indicators are estimated using the census data.

However, this method does not explicitly consider the trade-off between unbiasedness and efficiency when data are pooled. Therefore, I study a variation of the second method, which takes the variances of the estimates into consideration (henceforth abbreviated with the “compare MSE method”). I find that this method outperforms the other ones in making correct pooling decisions.

This paper is organized as follows. In section 2, I provide a brief description

of the ELL method. In section 3, the data used in this paper are discussed. In section 4, a simple model is studied to provide theoretical foundations of the possible gains and losses from pooling. In section 5, I discuss pooling based on the actual data and evaluate the reliability of a few methods in making pooling decisions. Section 7 concludes the paper.

2 Summary of the ELL Methodology

The idea of the ELL method⁴ is to first estimate the joint distribution of y_h , a variable on which the indicators of poverty are based, and a vector of variables x_h using a smaller and richer sample (e.g. data from a survey). By restricting x_h to be the variables on which a larger sample (e.g. data from a census) also provides information, the distribution of y_h for any sub-sample of the large sample can be generated by using the estimated distribution and the observed x_h in the larger sample. This generated distribution of y_h can then be used to generate the poverty indicators. The following is a brief summary of the method.

2.1 Consumption model

Consumption per capita is often used to measure poverty. An estimated joint distribution of consumption per capita y_h is obtained using the ELL method by developing a linear model of y_h on a vector of observed variables x_h :

$$\ln y_{ch} = x'_{ch}\beta + u_{ch}, \quad (1)$$

⁴This paragraph is drawn from Elbers et al. (2002b).

where y_{ch} is the household consumption per capita for household h in location c , x_{ch} is the vector of explanatory variables, and u_{ch} is an error term. It should be noted that this model is only used for predicting y_{ch} but not to measure the direct effect of x_{ch} on y_{ch} , so the endogeneity of the explanatory variables is not of concern here. As the results of this model are going to be used to predict y_{ch} in the census, it is preferred that the model fits most closely to the observations that represent a large part of the census population. Therefore population expansion factors are used as weights in this regression.

The residual term u_{ch} is defined as:

$$u_{ch} = \mu_c + \varepsilon_{ch}, \tag{2}$$

where μ_c is a location component, and ε_{ch} is a household component of the residual. The location component μ_c is used to capture the part of the error term which is due to the location characteristics common to all households in that location. The household component of the residual ε_{ch} reflects unobserved household characteristics which are not correlated with the location effect.

The variances of these two components of the error term reflect how much the household's predicted consumption deviates from its actual consumption. This deviation is one of the sources of the prediction error of the poverty indicators. The idiosyncratic component ε_{ch} falls approximately proportionately in sample size (Elbers et al. 2002a), so for a large enough sample the idiosyncratic component of the error term does not cause serious problems to the precision of the estimates of poverty indicators. The location component

μ_c does not fall in sample size, so it is important to capture as much of the location effect in the consumption model as possible. One way to do this is to calculate the means of the observed variables (e.g. average level of education) at certain location level (e.g. enumeration area (EA)) using the census data, insert these variables into the survey data and use them as regressors in the consumption model. These variables of census means can often do a good job in capturing the location effect.

This consumption model is estimated using Generalized Least Squares (GLS). An Ordinary Least Squares (OLS) estimation is first performed to obtain the variance-covariance matrix of the error term. The residuals \hat{u}_{ch} from the OLS estimation can be decomposed into two parts:

$$\hat{u}_{ch} = \hat{u}_c + (\hat{u}_{ch} - \hat{u}_c) = \hat{\mu}_c + e_{ch} \quad (3)$$

where a subscript “.” indicates an average over that index.

The variance of the location component $\hat{\sigma}_\mu^2$ can be estimated non-parametrically using $\hat{\mu}_c$. The component e_{ch} can be used to estimate the variances of ε_{ch} . A logistic form is used in this estimation:

$$\ln \left(\frac{e_{ch}^2}{A - e_{ch}^2} \right) = z'_{ch} \hat{\alpha} + r_{ch}, \quad (4)$$

where z_{ch} are the variables which best explain variation in e_{ch}^2 . In this way the prediction is bounded between zero and a maximum A . If A is set equal to $1.05 \times \max e_{ch}^2$ and $B = \exp(z'_{ch} \hat{\alpha})$, using the delta method the variance of

ε_{ch} is estimated as:

$$\hat{\sigma}_{ch}^2 = \left(\frac{AB}{A+B} \right) + \frac{1}{2} \text{Var}(\hat{r}) \left(\frac{AB(1-B)}{(1+B)^3} \right). \quad (5)$$

Once these two variances are calculated, they can be plugged into the variance-covariance matrix of the error term and the model can be estimated by GLS.

2.2 Poverty indicators

The second set of tasks in the ELL method is to apply the estimates from the regression of the consumption model to the census data, predict the consumption for the census data and calculate the poverty indicators.

This task is done by simulation. For each simulation a vector of the parameters $\tilde{\beta}$ is drawn from the multivariate normal distribution described by the GLS estimates of the consumption model and the associated variance-covariance matrix. The location component of the error term $\tilde{\mu}_c$ is drawn randomly with replacement from the set of $\hat{\mu}_c$. To draw the household component $\tilde{\varepsilon}_{ch}$, $\tilde{\varepsilon}_{ch}^*$ is first drawn for each household with replacement from the set of all standardized residuals⁵, or from the standard residuals that correspond to the cluster from which the household's location effect is derived. The household component is then set to $\tilde{\varepsilon}_{ch}^* \times \hat{\sigma}_{ch}$. For each simulation, with the drawn values of $\tilde{\beta}$, $\tilde{\mu}_c$, and $\tilde{\varepsilon}_{ch}$, the value of per capita consumption \hat{y}_{ch} is estimated as:

$$\hat{y}_{ch} = \exp(x'_{ch}\tilde{\beta} + \tilde{\mu}_c + \tilde{\varepsilon}_{ch}). \quad (6)$$

⁵Standardized residuals are calculated using the formula: $e^* = \frac{e_{ch}}{\hat{\sigma}_{ch}} - \left(\frac{1}{H} \sum_{ch} \frac{e_{ch}}{\hat{\sigma}_{ch}} \right)$.

Finally, the full vector of simulated consumption per capita \hat{y}_{ch} is used to calculate the mean and standard deviation of each poverty indicator.

3 The data

3.1 Census data

The General Population, Housing, and Establishment Census was conducted by the Central Statistical Organization, Ministry of Planning & International Cooperation, Republic of Yemen in December 2004. The total number of households covered in the census is 2,831,929. For urban households, the administration contains six levels: governorate, district, sub-district, city, zone and neighborhood. The administration for rural households contains five levels: governorate, district, sub-district, village and sub-village. All the administrative areas are then divided into 21,582 EAs.

Two kind of questionnaires are used in the census: the short questionnaire and the long questionnaire. The short questionnaire has seven components: housing unit properties, transport vehicles and durable goods, general & social data, data of disabled household members, married status and educational data. The long questionnaire is used for 10% of the households and it contains all the sections in the short one plus three sections: economic data, fertility data and mortality data. The long questionnaire provides richer information, but since it is only used by 10% of the households using the household level data from the long questionnaire often increases the standard errors of the estimates of the poverty indicators. In the case of Yemen

the three extra sections covered in the long questionnaire provide little common information compared to the survey data. Therefore they are not used in generating variables at the household level. However, the economic data provided by the long questionnaire can be useful in predicting consumption and they can be used in generating variables of census means about average economic status at a certain location.

3.2 Survey data

The Household Budget Survey (HBS) 2005-06 was also conducted by the Central Statistical Organization of Yemen. The sample frame for the HBS was the 2004 General Population, Housing, and Establishment Census. Yemen consists of 21 governorates. The study population was sorted into 38 strata. The 17 governorates were represented by two strata (urban and rural), whereas Sana'a City and Aden are only urban and Remah and Sana'a Region are only rural. This resulted in 19 urban strata and 19 rural strata⁶.

Within each stratum, the survey was selected in two stages. In the first stage, a certain number of Census EAs were selected with probability proportional to size (using as a measure of size the number of households according to the pre-census estimates available in January 2005). In the second stage, 12 households were picked from each EA by systematic equal probability sampling.

In order to produce estimates of consumption in all governorates of both rural and urban populations, the 1,200 EAs was distributed across strata by a combination of allocation proportional to size and equal allocation. The

⁶The description of the design of the HBS is drawn from Godoy and Muñoz (2006).

final sample allocation is as shown in Table 1.

The HBS data contain information on household roster, activities, dwelling conditions, health, education, anthropometrics, income, durable goods and consumption. Among these, information on household roster, dwelling conditions, education and durable goods is also available in the census.

4 The effects of pooling on the estimates of poverty indicators: Illustrations based on a simple model

4.1 A simple model

In this section, I study the effects of pooling on the estimates of poverty indicators using a simple model. By doing this, I illustrate the possible gains and losses from pooling.

4.1.1 Assumptions and estimates

Assume that the data from two strata are generated according to the following model.

$$y_1 = x_1\beta_1 + \varepsilon_1, \tag{7}$$

$$y_2 = x_2\beta_2 + \varepsilon_2, \tag{8}$$

where $\varepsilon_1 \sim N(0, \sigma_1^2)$ and $\varepsilon_2 \sim N(0, \sigma_2^2)$. The error terms ε_1 and ε_2 are independent and $E(x_1\varepsilon_1) = 0$, $E(x_2\varepsilon_2) = 0$. y_1 , x_1 and ε_1 are $N_1 \times 1$

matrices, y_2 , x_2 and ε_2 are $N_2 \times 1$ matrices.

If I pool the data from the two strata, the model becomes

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \beta_p + \varepsilon_p, \quad (9)$$

where $\varepsilon_p \sim N(0, \sigma_p^2)$.

For the estimate of β_p I get:

$$\begin{aligned} \hat{\beta}_p &= \frac{\sum_{i=1}^{N_1} x_{1i} y_{1i} + \sum_{j=1}^{N_2} x_{2j} y_{2j}}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} \\ &= \frac{\sum_{i=1}^{N_1} x_{1i} (x_{1i} \beta_1 + \varepsilon_{1i}) + \sum_{j=1}^{N_2} x_{2j} (x_{2j} \beta_2 + \varepsilon_{2j})}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} \\ &= \frac{\sum_{i=1}^{N_1} x_{1i}^2}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} \beta_1 + \frac{\sum_{j=1}^{N_2} x_{2j}^2}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} \beta_2 \\ &\quad + \frac{\sum_{i=1}^{N_1} x_{1i} \varepsilon_{1i} + \sum_{j=1}^{N_2} x_{2j} \varepsilon_{2j}}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} \end{aligned} \quad (10)$$

It follows that

$$E(\hat{\beta}_p) = \frac{\sum_i^{N_1} x_{1i}^2}{\sum_i^{N_1} x_{1i}^2 + \sum_j^{N_2} x_{2j}^2} \beta_1 + \frac{\sum_j^{N_2} x_{2j}^2}{\sum_i^{N_1} x_{1i}^2 + \sum_j^{N_2} x_{2j}^2} \beta_2, \quad (11)$$

$$Var(\hat{\beta}_p) = \frac{\sum_i^{N_1} x_{1i}^2}{(\sum_i^{N_1} x_{1i}^2 + \sum_j^{N_2} x_{2j}^2)^2} \sigma_1^2 + \frac{\sum_j^{N_2} x_{2j}^2}{(\sum_i^{N_1} x_{1i}^2 + \sum_j^{N_2} x_{2j}^2)^2} \sigma_2^2. \quad (12)$$

4.1.2 The expectation

Looking at the expectation of the estimate, I have

$$|E(\hat{\beta}_p) - \beta_1| = \frac{\sum_{j=1}^{N_2} x_{2j}^2}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} |\beta_1 - \beta_2| \quad (13)$$

$$|E(\hat{\beta}_p) - \beta_2| = \frac{\sum_{i=1}^{N_1} x_{1i}^2}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2} |\beta_1 - \beta_2| \quad (14)$$

Thus $\hat{\beta}_p$ is a biased estimate of both β_1 and β_2 if $\beta_1 \neq \beta_2$. The bias is determined by the difference between β_1 and β_2 scaled by $\frac{\sum_{j=1}^{N_2} x_{2j}^2}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2}$ or $\frac{\sum_{i=1}^{N_1} x_{1i}^2}{\sum_{i=1}^{N_1} x_{1i}^2 + \sum_{j=1}^{N_2} x_{2j}^2}$.

To get a sense of the effects of the properties of x_1 and x_2 on the bias, I use the approximation

$$\sum_t^N x_t^2 \approx N * (\mathbb{E}(x))^2 + N * \text{Var}(x).$$

Then it follows from equation (13) that

$$\begin{aligned} |E(\hat{\beta}_p) - \beta_1| &= \frac{1}{1 + \frac{\sum_{i=1}^{N_1} x_{1i}^2}{\sum_{j=1}^{N_2} x_{2j}^2}} |\beta_1 - \beta_2|, \\ &= \frac{1}{1 + \frac{N_1(\mathbb{E}(x_1))^2 + N_1 \text{Var}(x_1)}{N_2(\mathbb{E}(x_2))^2 + N_2 \text{Var}(x_2)}} |\beta_1 - \beta_2|, \\ &= \frac{1}{1 + \frac{N_1(\mathbb{E}(x_1))^2 + \text{Var}(x_1)}{N_2(\mathbb{E}(x_2))^2 + \text{Var}(x_2)}} |\beta_1 - \beta_2|. \end{aligned} \quad (15)$$

Similarly

$$|E(\hat{\beta}_p) - \beta_2| = \frac{1}{1 + \frac{N_2(\mathbb{E}(x_2))^2 + \text{Var}(x_2)}{N_1(\mathbb{E}(x_1))^2 + \text{Var}(x_1)}} |\beta_1 - \beta_2|. \quad (16)$$

If weighting is used in the regressions and the weights are the same for observations in the same stratum (N_{c1} and N_{c2} are the number of observations in the census in stratum 1 and 2 respectively), then,

$$\begin{aligned} |\mathbb{E}(\hat{\beta}_p) - \beta_1| &= \frac{1}{1 + \frac{W_1 N_1 (\mathbb{E}(x_1))^2 + \text{Var}(x_1)}{W_2 N_2 (\mathbb{E}(x_2))^2 + \text{Var}(x_2)}}} |\beta_1 - \beta_2|, \\ &= \frac{1}{1 + \frac{N_{c1} (\mathbb{E}(x_1))^2 + \text{Var}(x_1)}{N_{c2} (\mathbb{E}(x_2))^2 + \text{Var}(x_2)}}} |\beta_1 - \beta_2|. \end{aligned} \quad (17)$$

Similarly,

$$|\mathbb{E}(\hat{\beta}_p) - \beta_2| = \frac{1}{1 + \frac{N_{c2} (\mathbb{E}(x_2))^2 + \text{Var}(x_2)}{N_{c1} (\mathbb{E}(x_1))^2 + \text{Var}(x_1)}}} |\beta_1 - \beta_2|. \quad (18)$$

The bias is bigger for the stratum with less observations. Thus it is better to pool the strata with similar number of observations.

4.1.3 The variance

Looking at the variance, I have

$$\text{Var}(\hat{\beta}_p) = \frac{\sum_i^{N1} x_{1i}^2}{(\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2)^2} \sigma_1^2 + \frac{\sum_i^{N2} x_{2j}^2}{(\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2)^2} \sigma_2^2.$$

If $\sigma_1 = \sigma_2 = \sigma$,

$$\text{Var}(\hat{\beta}_p) = \frac{\sigma^2}{\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2}.$$

If I do not pool the data,

$$\begin{aligned} \text{Var}(\hat{\beta}_1) &= \frac{\sigma^2}{\sum_i^{N1} x_{1i}^2}, \\ \text{Var}(\hat{\beta}_2) &= \frac{\sigma^2}{\sum_j^{N2} x_{2j}^2}. \end{aligned}$$

I have $\text{Var}(\hat{\beta}_p) \leq \text{Var}(\hat{\beta}_1)$ and $\text{Var}(\hat{\beta}_p) \leq \text{Var}(\hat{\beta}_2)$. Thus if $\sigma_1 = \sigma_2 = \sigma$ and $\beta_1 = \beta_2$, pooling can produce unbiased and more efficient estimates. However, if $\sigma_1 \neq \sigma_2$, the situation is different. In terms of efficiency, I can look at the ratio of $\text{Var}(\hat{\beta}_p)$ over $\text{Var}(\hat{\beta}_1)$ and the ratio of $\text{Var}(\hat{\beta}_p)$ over $\text{Var}(\hat{\beta}_2)$:

$$\frac{\text{Var}(\hat{\beta}_p)}{\text{Var}(\hat{\beta}_1)} = \left(\frac{\sum_i^{N1} x_{1i}^2 \sigma_1^2}{(\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2)^2} + \frac{\sum_i^{N2} x_{2j}^2 \sigma_2^2}{(\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2)^2} \right) \frac{\sum_i^{N1} x_{1i}^2}{\sigma_1^2}, \quad (19)$$

$$\frac{\text{Var}(\hat{\beta}_p)}{\text{Var}(\hat{\beta}_2)} = \left(\frac{\sum_i^{N1} x_{1i}^2 \sigma_1^2}{(\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2)^2} + \frac{\sum_i^{N2} x_{2j}^2 \sigma_2^2}{(\sum_i^{N1} x_{1i}^2 + \sum_j^{N2} x_{2j}^2)^2} \right) \frac{\sum_j^{N2} x_{2j}^2}{\sigma_2^2}. \quad (20)$$

It can be seen from these two equations that $\frac{\text{Var}(\hat{\beta}_p)}{\text{Var}(\hat{\beta}_1)}$ is increasing in $\frac{\sigma_2^2}{\sigma_1^2}$ and $\frac{\text{Var}(\hat{\beta}_p)}{\text{Var}(\hat{\beta}_2)}$ is increasing in $\frac{\sigma_1^2}{\sigma_2^2}$. Thus the stratum with smaller σ is more likely to lose its efficiency when data are pooled.

It can be derived that if

$$\frac{\sigma_2^2}{\sigma_1^2} > 2 + \frac{\sum_j^{N2} x_{2j}^2}{\sum_i^{N1} x_{1i}^2}, \quad (21)$$

then $\text{Var}(\hat{\beta}_p) > \text{Var}(\hat{\beta}_1)$, which means that stratum 1 loses its efficiency from pooling.

If

$$\frac{\sigma_2^2}{\sigma_1^2} < \frac{\sum_i^{N_2} x_{2j}^2}{2 \sum_j^{N_2} x_{2j}^2 + \sum_i^{N_1} x_{1i}^2}, \quad (22)$$

then $\text{Var}(\hat{\beta}_p) > \text{Var}(\hat{\beta}_2)$, which means that stratum 2 loses its efficiency from pooling.

4.2 Illustrations based on the simple model

From the analysis above, several parameters, which may determine if pooling should be preferred, are detected: $|\beta_1 - \beta_2|$, $\frac{N_{c1}}{N_{c2}}$ and $\frac{\sigma_1^2}{\sigma_2^2}$. To illustrate how pooling works under different sets of parameters, simulations are used to draw the density functions of the estimates of the headcount indicator. The steps of the data generation and simulation can be found in Appendix A.

In the first setting, all the parameters used in the simulation are equal in the two strata. This simulation is used to illustrate the gain of efficiency from pooling. Figure 1 shows the density function of the estimates of the headcount of the two strata⁷. The straight lines indicate the true values. It can be seen that both the estimates from the pooled and non-pooled data are unbiased but the estimates from the pooled data are more efficient. More important is that the estimates from the pooled data have thinner tails which means that estimates from the pooled data have a much smaller probability

⁷The poverty line is chosen to make the headcount rate to be around 0.5. I did this simulation for different values of the poverty line. The graph remains similar.

to be very far away from their true values.

In the next setting, β_2 is set to be different from β_1 and the rest of the parameters are unchanged. Figure 2 shows the density functions in this setting. This figure shows the trade-off between unbiasedness and efficiency when pooled data are used. As in Figure 1, the variances of the estimates are smaller using the pooled data than that of using the non-pooled data. However, the estimates from the pooled data are biased. The true value of the headcount lies in between of the estimate of headcount of the first stratum and that of the second stratum, as predicted by equation (11).

In the third setting, σ_2 is set to be different from σ_1 and the rest of the parameters are the same as in Figure 1. Figure 3 shows the density functions in this setting. It can be seen from this figure that the estimates from the pooled data are biased, and for the stratum with smaller σ the estimate loses its efficiency compared to that from the non-pooled data. This confirms what equations (19) and (20) show: the pooling estimates of the stratum with the smaller variance are more likely to lose its advantage of efficiency.

In the last setting, I look at the effects of $\frac{N_{c1}}{N_{c2}}$. If all the other parameters are the same for the two strata, making N_{c2} different from N_{c1} will not generate a different figure other than Figure 1. Thus in this setting I make N_{c1} different from N_{c2} and β_1 different from β_2 . Figure 4 shows the density function in this setting. As it was concluded from equations (17) and (18), the estimates from the pooled data are much more biased for the stratum with fewer observations in the census.

From this simple model, it can already be seen that to enjoy the advantage of pooling, it is better to pool the strata with similar β , σ and N_c . In reality,

β and σ are not known and models are much more complicated than the one discussed in this section. Section 5 discusses pooling in the context of actual data.

5 Pooling in the context of actual data

Section 4 shows that whether data from different strata should be pooled depends on various properties of the data even in a very simple model. The situation can only be more complicated in the context of actual data. In this section, I propose three methods which can be conducted to make pooling decisions while producing poverty maps. I will first describe the methods in section 5.1 and in section 5.2 I will design a simulation procedure to test how reliable the methods are. Section 5.3 shows the results of the simulation.

5.1 Methods to make pooling decisions

What I have when producing poverty maps is first the survey data which include consumption data. With the consumption data, I can build the consumption models discussed in section 2.1 and I can also calculate the headcount of the survey data H_s ⁸. Second, I also have the census data which do not contain consumption data, so the headcount of the census data (denoted as H_c) is not known.

Let's consider how would we make pooling decisions if there were consumption data in the census. The aim of the ELL method is to estimate H_c . A

⁸I use in this section only the headcount indicator to measure poverty but all the methods discussed in this section can be applied to other poverty indicators as well.

good estimator should be unbiased and efficient. An obvious method to decide whether to pool the data or not is to estimate H_c using first a pooled consumption model then a non-pooled consumption model built using the survey data. To take both unbiasedness and efficiency into account, MSE's of the two estimates of H_c are calculated and compared. The decision should be based on whether using the pooled data or the non-pooled data minimizes the MSE. The details of this method can be found in Table 2 under the method name "Minimize MSE".

In reality H_c is not known, so the method "Minimize MSE" cannot be conducted. I therefore propose three methods using the available information (survey data with consumption data and census data without consumption data) to decide whether to pool the data or not. The details of the three methods can be found in Table 2. In the first method ("Survey Map"), the pooled and the non-pooled consumption models built using the survey data are applied to the observations in the survey. In this way I can get the estimates of headcount of the survey H_s from both the pooled data and the non-pooled data. I then compare the MSE's of these two estimates to make pooling decisions. This method only uses the survey data to make pooling decisions but it does look at the estimates of the headcount instead of only looking at the coefficients in the consumption model.

In the second method ("Compare Value"), I apply the pooled and the non-pooled consumption models built using the survey data to the observations in the census. If the two estimates of H_c are similar, the data are pooled; otherwise, the data are not pooled. This method looks at the estimates of the headcount as the "survey map method" but it uses both survey data and

census data to make pooling decisions⁹.

The third method (“compare MSE”) is similar to the second one. The only difference is that instead of comparing the estimates themselves it tries to calculate the MSE’s of the estimates. Since the true value of H_c is not known, the MSE’s are calculated using H_s . In this way, the tradeoff between biasedness and efficiency of the estimate is taken into account.

5.2 Simulation design

To test the reliability of the three methods proposed in section 5.1, I design a simulation set-up. As explained in section 2, the idea of the ELL method is to use the survey, which is a representative sample of the census, to predict the consumption per capita and poverty indicators in the census. In this simulation set-up I draw different representative samples from the census data. These samples can then be used to build consumption models and to predict poverty indicators using the census data.

Since consumption per capita is usually not available in the census, a consumption model is first built based on the survey data. Consumption per capita in the census is calculated based on the estimates of the coefficients and the variances of the error terms in the consumption model, which are built stratum by stratum. Then samples can be drawn from the census with constructed consumption data.

The constructed consumption of the census can be used later to obtain the “true” decision of pooling. The “true decision” can then be compared to the

⁹The consumption models are built using the survey data and the models are applied to the census data to estimate the headcount H_c .

decisions made by using the three methods discussed in section 5.1 to check if the three methods are reliable.

I apply the simulation procedure to two strata from the Yemen data (rural Abyan and rural Shabwah). Geographically, Abyan shares a border with Shabwah. This makes it not unreasonable that data from the two strata can be pooled. The survey covers a relatively small number of households in these strata (rural Abyan 216 and rural Shabwah 180). This makes it favorable to pool the data with the purpose to save time in producing poverty maps and the hope that pooling will generate more efficient estimates of poverty indicators.

The steps of the simulation are:

1. Generate consumption per capita for the census data
 - Take data from two strata from the census and the survey.
 - Use the survey data to estimate the coefficients in the consumption model and the variances of the error terms. The consumption models are built stratum by stratum. Consumption per capita data for the census data are generated based on the estimates from the two stratum by stratum models.
2. Generate sample data
 - Sample N_1 observations from the census stratum 1 to be sample stratum 1. This is done in two stages. First a certain number of EA's are sampled with probabilities based on the size of the EA. In the second stage 12 households¹⁰ are sampled from each EA.

¹⁰In the HBS data from Yemen, 12 households were sampled in each EA.

These households are sampled with equal probability.

- Sample N_2 observations from the census stratum 2 to be sample stratum 2 in the same way as sampling sample stratum 1.

3. Choose the sample¹¹

- Repeat step 2 100 times to get 100 samples for both stratum 1 and stratum 2.
- Calculate the headcount indicators for the 100 samples generated.
- Calculate the means and the variances of the headcount indicators for each stratum.
- Group the samples into 5 groups for each stratum and calculate the means and the variances of the headcount indicators for each group.
- Choose one group for each stratum with the mean of the headcount indicators close to its true value and the variance of the headcount indicators close to its value calculated using the 100 samples.
- Use the 20 samples for each stratum to do the later analysis.

4. Estimate consumption models using the samples

- Non-pooled: Estimate coefficients in the consumption model and variance of the error terms using sample stratum 1 chosen in step

¹¹The reason I use this step to choose 20 samples from the 100 samples is that step 4 are very time consuming. It is not possible to repeat them on 100 samples. Therefore, I use this step to choose 20 samples with the mean of the headcount indicator close to its true value and the variance close to that calculated using the 100 samples. The means and the variances of the headcount indicators of every 20 samples are reported in Table B1. The samples chosen are samples 1-20 for stratum 1 and samples 61-70 for stratum 2.

3¹².

- Non-pooled: Estimate coefficients in the consumption model and variance of the error terms using sample stratum 2 chosen in step 3.
- Pooled¹³: Estimate coefficients in the consumption model and variance of the error terms based on both sample stratum 1 and sample stratum 2.

5. Repeat step 4 19 times

5.3 Simulation results

Using the 20 samples I draw, I first conduct the method “Minimize MSE” to obtain the “true” decisions. The results are shown in Table 3 and Table 4 with the mean, standard error and MSE of the estimates of the headcount for both pooled data and non-pooled data for rural Abyan and rural Shabwah respectively. After comparing the MSE of the pooled data to that of the non-pooled data, I can know the decisions whether to pool the data or not to achieve minimum MSE. The decisions are listed in the last column. These decisions will be used later as a benchmark to check if the other methods can be used to make correct decisions.

I then conduct the three methods proposed in section 5.1. The results of the “survey map method” are reported in Table 5 and Table 6 and the results

¹²The estimation is done in the PovMap program. The explanatory variables are selected using stepwise estimation by the program.

¹³Pooled model means that observations from the two strata are used and stratum dummies are included in the candidates of the variables in the consumption model.

of the “compare MSE method” are reported in Table 7 and Table 8. The decisions from these two methods are listed in the last column of each table. The decisions from the “compare value method” can be obtained from Table 7 and Table 8 as well by comparing the column “Non-pooled- \hat{H}_c^p ” and column “Pooled- \hat{H}_c^{np} ”.

Table 9 and Table 10 summarize the results. The samples are ordered by the difference between H_c and H_s . Thus from index 1 to index 20 the value of the poverty indicator of the sample H_s goes farther and farther away from its true value H_c and the sample is less and less representative. The decisions obtained from the “survey map method” are reported in column “Survey Map Method”. In general, this method does not work very well especially not for rural Shabwah. The successful rate is 50% compared to column “Minimize MSE”. This method does not seem to work better for the more representative samples.

The decisions obtained from the “compare value method” are reported in column “Compare Value Method”. If the difference between the pooled estimate and the non-pooled estimate in Table 7 and Table 8 is smaller than 3%¹⁴, I consider that the estimates are similar. Comparing these decisions to the decisions which minimize the MSE’s, I find that the successful rate of this method is not very high (50%-70%). Table 9 and Table 10 also show that this method does not work better for the 10 samples which are more representative.

The decisions obtained using the “compare MSE method” are shown in col-

¹⁴There is no particular reason to choose 3%. As the estimates are in the range from 40% to 60%, a difference of 3% makes them look similar.

umn “compare MSE Method” in Table 9 and Table 10. As the decision which minimizes MSE is based on the MSE calculated using the true value of the headcount indicator H_c , the closer the value of the survey H_s to the true value H_c the better this method works. This is proved by the results from Table 9 and Table 10. If 20 samples are considered, the method gives the right decisions for 85% and 65% of the samples of rural Abyan and rural Shabwah respectively. If only the first 10 samples are considered, the method gives the right decisions for 90% and 80% of the samples of rural Abyan and rural Shabwah respectively. This method works better than the other methods and it works especially well when the samples are representative.

6 Conclusion

Whether to pool data from different strata or not is an unresolved issue in poverty mapping. This paper starts with a very simple model to show that the general belief that pooling produces more efficient but more biased estimates is only true in some special cases. Whether pooling is beneficial depends on many properties of the data even in the simple model.

The question I study in this paper is not whether pooling is good. It certainly depends on the data and the model. What I focus on is when pooling is better and how to detect it. Many factors may determine whether data from different strata can be pooled. For example, the coefficients and the variances of the error terms in the consumption model, the properties of the survey data and the properties of the census data. An effective method to

be employed to make pooling decisions needs to take these factors into consideration. I use simulation studies to check the reliability of three methods to make pooling decisions. I find that one method in general works better than the other ones. This method can be conducted by researchers producing poverty maps to make pooling decisions.

Appendix A: Steps of simulations using the simple model

1. Generate Census Data

- Choose values for $E(x)$, $\text{Var}(x)$, σ_1 , σ_2 , β_1 , β_2 , N_{s1} , N_{s2} , N_{c1} and N_{c2}
- Generate N_{c1} samples of x_{c1} from a normal distribution with mean $E(x)$ and variance $\text{Var}(x)$, and ε_{c1} from a normal distribution with mean zero and variance σ_1^2
- From this calculate y_{c1} , using $y_{c1} = \beta_1 x_{c1} + \varepsilon_{c1}$
- Generate N_{c2} samples of x_{c2} from a normal distribution with mean $E(x)$ and variance $\text{Var}(x)$, and ε_{c2} from a normal distribution with mean zero and variance σ_2^2
- From this calculate y_{c2} , using $y_{c2} = \beta_2 x_{c2} + \varepsilon_{c2}$

2. Generate Survey Data

- Randomly draw N_{s1} (y_{c1}, x_{c1}) pairs from the N_{c1} pairs to be survey stratum 1: (y_{s1}, x_{s1})
- Randomly draw N_{s2} (y_{c2}, x_{c2}) pairs from the N_{c2} pairs to be survey stratum 2: (y_{s2}, x_{s2})

3. Estimate betas for surveys

- Non-pooled: Estimate β_1 from $y_{s1} = \hat{\beta}_1 x_{s1} + \varepsilon_{s1}$, estimate σ_1 from $\frac{\sum_{i=1}^{N1} (y_{s1i} - \hat{\beta}_1 x_{s1i})^2}{N1-1}$ and estimate the standard error of $\hat{\beta}_1$ from $\hat{\sigma}_{\hat{\beta}_1} = \frac{\hat{\sigma}_1}{\sum_{i=1}^{N_{s1}} x_{s1i}^2}$
- Non-pooled: Estimate β_2 from $y_{s2} = \hat{\beta}_2 x_{s2} + \varepsilon_{s2}$, estimate σ_2 from $\frac{\sum_{i=1}^{N2} (y_{s2i} - \hat{\beta}_2 x_{s2i})^2}{N2-1}$ and estimate the standard error of $\hat{\beta}_2$ from $\hat{\sigma}_{\hat{\beta}_2} = \frac{\hat{\sigma}_2}{\sum_{i=1}^{N_{s2}} x_{s2i}^2}$
- Pooled: Estimate β_p from $(y_{s1}|y_{s2}) = \hat{\beta}_p(x_{s1}|x_{s2}) + (\varepsilon_{s1}|\varepsilon_{s2})$, estimate σ_p from $\frac{\sum_{i=1}^{N1+N2} ((y_{s1}|y_{s2})_i - \hat{\beta}_p(x_{s1}|x_{s2})_i)^2}{N1+N2-1}$ and estimate the standard error of $\hat{\beta}_p$ from $\hat{\sigma}_{\hat{\beta}_p} = \frac{\hat{\sigma}_p}{\sum_{i=1}^{N_{s1}+N_{s2}} (x_{s1}|x_{s2})_i^2}$

4. Calculate poverty indicators by estimating y for census

- Set the poverty line: the same number for all the households
- Non-pooled: Calculate poverty indicator PI^{c1} by simulating $\hat{y}_{c1} = \tilde{\beta}_1 x_{c1} + \tilde{\varepsilon}_{c1}$, where $\tilde{\beta}_1$ is 100 draws from $N(\hat{\beta}_1, \hat{\sigma}_{\hat{\beta}_1})$ and $\tilde{\varepsilon}_{c1}$ is 100 draws from $N(0, \hat{\sigma}_1)$
- Non-pooled: Calculate poverty indicator PI^{c2} by simulating $\hat{y}_{c2} = \tilde{\beta}_2 x_{c2} + \tilde{\varepsilon}_{c2}$, where $\tilde{\beta}_2$ is 100 draws from $N(\hat{\beta}_2, \hat{\sigma}_{\hat{\beta}_2})$ and $\tilde{\varepsilon}_{c2}$ is 100 draws from $N(0, \hat{\sigma}_2)$

- Pooled: Calculate PI^{c1p} by simulating $\hat{y}_{c1p} = \tilde{\beta}_p x_{c1} + \tilde{\varepsilon}_{c1p}$, where $\tilde{\beta}_p$ is 100 draws from $N(\hat{\beta}_p, \hat{\sigma}_{\hat{\beta}_p})$ and $\tilde{\varepsilon}_{c1p}$ is 100 draws from $N(0, \hat{\sigma}_p)$
- Pooled: Calculate PI^{c2p} by simulating $\hat{y}_{c2p} = \tilde{\beta}_p x_{c2} + \tilde{\varepsilon}_{c2p}$, where $\tilde{\beta}_p$ is 100 draws from $N(\hat{\beta}_p, \hat{\sigma}_{\hat{\beta}_p})$ and $\tilde{\varepsilon}_{c2p}$ is 100 draws from $N(0, \hat{\sigma}_p)$

5. Repeat steps 2-4 299 times

Appendix B

Table B1
Results: Choose the Samples

	Rural Abyan		Rural Shabwah	
	mean	variance	mean	variance
Sample 1-20	0.5021	0.0033	0.5254	0.0125
Sample 21-40	0.4990	0.0020	0.5111	0.0047
Sample 41-60	0.4883	0.0036	0.5273	0.0045
Sample 61-80	0.5018	0.0048	0.5277	0.0061
Sample 81-100	0.5060	0.0029	0.5290	0.0033
Sample 1-100	0.4994	0.0032	0.5241	0.0060
True value H_c	0.5002		0.5268	

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Table 1: Number of households in the census data and HBS final sample allocation

	No. of households (census)			No. of clusters (survey)			No. of households (survey)		
	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Governorate									
11 Ibb	54,126	259,492	313,618	43	41	84	516	492	1,008
12 Abyan	15,524	43,446	58,970	30	18	48	360	216	576
13 Sana'a City	260,825	4,971	265,796	156	0	156	1,872	0	1,872
14 Al-Baida	14,023	55,774	69,797	29	19	48	348	228	576
15 Taiz	88,474	304,262	392,736	56	40	96	672	480	1152
16 Al-Jawf	7,236	49,230	56,466	22	14	36	264	168	432
17 Hajja	17,275	169,586	186,861	30	30	60	360	360	720
18 Al-Hodeida	120,603	246,919	367,522	75	33	108	900	396	1,296
19 Hadramout	66,375	75,605	141,980	41	19	60	492	228	720
20 Dhamar	25,879	173,069	198,948	31	29	60	372	348	720
21 Shabwah	9,637	43,412	53,049	21	15	36	252	180	432
22 Sa'adah	12,924	68,529	81,453	28	20	48	336	240	576
23 Sana'a Region	3,653	112,119	115,772	0	24	24	0	288	288
24 Aden	97,289	0	97,289	72	0	72	864	0	864
25 Laheg	9,720	104,882	114,602	25	23	48	300	276	576
26 Mareb	3,962	24,029	27,991	22	14	36	264	168	432
27 Al-Mahweet	4,674	60,849	65,523	27	21	48	324	252	576
28 Al-Maharh	5,220	7,636	12,856	12	12	24	144	144	288
29 Amran	18,728	80,408	99,136	27	21	48	324	252	576
30 Al-Dhale	8,445	52,640	61,085	22	14	36	264	168	432
31 Remah	617	49,862	50,479	0	24	24	0	288	288
Total	845,209	1,986,720	2,831,929	769	431	1,200	9,228	5,172	14,400

Source.—No. of households in the census from author's calculation; HBS final sample allocation from Figure 3 in Godoy and Muñoz (2006).

Table 2: Description of the Methods

	Minimize MSE (benchmark)	Survey Map (method 1)
Input	survey with y (H_s known) census with y (H_c known)	survey with y (H_s known) census without y (H_c unknown)
Model	pooled (built using survey) non-pooled (built using survey)	pooled (built using survey) non-pooled (built using survey)
Output	\hat{H}_c^p and $\text{var}(\hat{H}_c^p)$ \hat{H}_c^{np} and $\text{var}(\hat{H}_c^{np})$	\hat{H}_s^p and $\text{var}(\hat{H}_s^p)$ \hat{H}_s^{np} and $\text{var}(\hat{H}_s^{np})$
Judgment	compare $MSE_c^p = (\hat{H}_c^p - H_c)^2 + \text{var}(\hat{H}_c^p)$ and $MSE_c^{np} = (\hat{H}_c^{np} - H_c)^2 + \text{var}(\hat{H}_c^{np})$	compare $MSE_s^p = (\hat{H}_s^p - H_s)^2 + \text{var}(\hat{H}_s^p)$ and $MSE_s^{np} = (\hat{H}_s^{np} - H_s)^2 + \text{var}(\hat{H}_s^{np})$
	Compare Value (method 2)	compare MSE (method 3)
Input	survey with y (H_s known) census without y (H_c unknown)	survey with y (H_s known) census without y (H_c unknown)
Model	pooled (built using survey) non-pooled (built using survey)	pooled (built using survey) non-pooled (built using survey)
Output	\hat{H}_c^p and $\text{var}(\hat{H}_c^p)$ \hat{H}_c^{np} and $\text{var}(\hat{H}_c^{np})$	\hat{H}_c^p and $\text{var}(\hat{H}_c^p)$ \hat{H}_c^{np} and $\text{var}(\hat{H}_c^{np})$
Judgment	compare \hat{H}_c^p and \hat{H}_c^{np}	compare $MSE_c^p = (\hat{H}_c^p - H_s)^2 + \text{var}(\hat{H}_c^p)$ and $MSE_c^{np} = (\hat{H}_c^{np} - H_s)^2 + \text{var}(\hat{H}_c^{np})$

Note.—In this table H means headcount, y means consumption, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively.

Table 3: Results of the Minimize MSE Method: Rural Abyan

	True Value	Survey Value	Non-Pooled			Pooled			Minimize MSE
	H_c	H_s	\hat{H}_c^{np}	$\sqrt{\text{var}(\hat{H}_c^{np})}$	$\sqrt{MSE_c^{np}}$	\hat{H}_c^p	$\sqrt{\text{var}(\hat{H}_c^p)}$	$\sqrt{MSE_c^p}$	
Sample 1	0.500	0.522	0.532	0.015	0.035	0.508	0.032	0.033	pool
Sample 2	0.500	0.531	0.508	0.015	0.016	0.492	0.021	0.022	do not pool
Sample 3	0.500	0.472	0.492	0.027	0.028	0.500	0.014	0.014	pool
Sample 4	0.500	0.534	0.519	0.022	0.028	0.474	0.051	0.057	do not pool
Sample 5	0.500	0.481	0.468	0.029	0.043	0.539	0.023	0.045	do not pool
Sample 6	0.500	0.446	0.434	0.033	0.073	0.427	0.011	0.074	do not pool
Sample 7	0.500	0.524	0.522	0.033	0.040	0.557	0.053	0.078	do not pool
Sample 8	0.500	0.444	0.407	0.050	0.106	0.445	0.061	0.082	pool
Sample 9	0.500	0.355	0.419	0.093	0.123	0.393	0.046	0.116	pool
Sample 10	0.500	0.528	0.571	0.038	0.080	0.510	0.032	0.033	pool
Sample 11	0.500	0.588	0.519	0.029	0.034	0.575	0.026	0.080	do not pool
Sample 12	0.500	0.570	0.526	0.040	0.048	0.449	0.055	0.075	do not pool
Sample 13	0.500	0.569	0.516	0.030	0.034	0.546	0.065	0.079	do not pool
Sample 14	0.500	0.491	0.468	0.022	0.038	0.490	0.011	0.015	pool
Sample 15	0.500	0.488	0.497	0.071	0.071	0.539	0.070	0.080	do not pool
Sample 16	0.500	0.467	0.439	0.040	0.074	0.488	0.020	0.024	pool
Sample 17	0.500	0.432	0.494	0.031	0.031	0.457	0.021	0.048	do not pool
Sample 18	0.500	0.519	0.494	0.015	0.016	0.476	0.034	0.042	do not pool
Sample 19	0.500	0.493	0.494	0.079	0.079	0.508	0.023	0.025	pool
Sample 20	0.500	0.588	0.593	0.028	0.097	0.608	0.027	0.111	do not pool

Note.—In this table H means headcount, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively. Table 2 column “Minimize MSE (benchmark)” shows how the MSE’s in this table are calculated.

Table 4: Results of the Minimize MSE Method: Rural Shabwah

	True Value	Survey Value	Non-Pooled			Pooled			Minimize MSE
	H_c	H_s	\hat{H}_c^{np}	$\sqrt{\text{var}(\hat{H}_c^{np})}$	$\sqrt{MSE_c^{np}}$	\hat{H}_c^p	$\sqrt{\text{var}(\hat{H}_c^p)}$	$\sqrt{MSE_c^p}$	
Sample 1	0.527	0.592	0.550	0.041	0.047	0.549	0.031	0.038	pool
Sample 2	0.527	0.528	0.576	0.032	0.059	0.534	0.019	0.020	pool
Sample 3	0.527	0.606	0.607	0.021	0.083	0.578	0.016	0.054	pool
Sample 4	0.527	0.453	0.438	0.051	0.102	0.466	0.032	0.069	pool
Sample 5	0.527	0.551	0.597	0.021	0.073	0.544	0.018	0.025	pool
Sample 6	0.527	0.492	0.497	0.027	0.040	0.490	0.010	0.038	pool
Sample 7	0.527	0.449	0.440	0.022	0.090	0.491	0.027	0.045	pool
Sample 8	0.527	0.366	0.371	0.051	0.164	0.426	0.065	0.120	pool
Sample 9	0.527	0.462	0.511	0.035	0.038	0.512	0.028	0.031	pool
Sample 10	0.527	0.459	0.390	0.053	0.147	0.408	0.028	0.122	pool
Sample 11	0.527	0.681	0.652	0.023	0.127	0.593	0.027	0.071	pool
Sample 12	0.527	0.463	0.473	0.019	0.057	0.436	0.051	0.104	do not pool
Sample 13	0.527	0.564	0.553	0.034	0.043	0.444	0.037	0.091	do not pool
Sample 14	0.527	0.577	0.523	0.016	0.016	0.507	0.011	0.022	do not pool
Sample 15	0.527	0.512	0.532	0.013	0.014	0.491	0.040	0.054	do not pool
Sample 16	0.527	0.514	0.524	0.010	0.011	0.525	0.018	0.018	do not pool
Sample 17	0.527	0.463	0.471	0.028	0.062	0.459	0.019	0.070	do not pool
Sample 18	0.527	0.607	0.570	0.024	0.049	0.552	0.024	0.035	pool
Sample 19	0.527	0.574	0.537	0.036	0.037	0.599	0.018	0.074	do not pool
Sample 20	0.527	0.640	0.600	0.048	0.088	0.611	0.020	0.087	pool

Note.—In this table H means headcount, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively. Table 2 column “Minimize MSE (benchmark)” shows how the MSE’s in this table are calculated.

Table 5: Results of the Survey Map Method: Rural Abyan

	True Value	Survey Value	Non-Pooled			Pooled			Survey Map
	H_c	H_s	\hat{H}_s^{np}	$\sqrt{\text{var}(\hat{H}_s^{np})}$	$\sqrt{MSE_s^{np}}$	\hat{H}_s^p	$\sqrt{\text{var}(\hat{H}_s^p)}$	$\sqrt{MSE_s^p}$	
Sample 1	0.500	0.522	0.521	0.042	0.042	0.523	0.057	0.057	do not pool
Sample 2	0.500	0.531	0.545	0.032	0.035	0.543	0.043	0.044	do not pool
Sample 3	0.500	0.472	0.490	0.038	0.042	0.506	0.037	0.050	do not pool
Sample 4	0.500	0.534	0.567	0.047	0.057	0.488	0.066	0.081	do not pool
Sample 5	0.500	0.481	0.472	0.059	0.060	0.523	0.044	0.061	do not pool
Sample 6	0.500	0.446	0.442	0.057	0.057	0.428	0.045	0.049	pool
Sample 7	0.500	0.524	0.483	0.048	0.063	0.506	0.074	0.076	do not pool
Sample 8	0.500	0.444	0.451	0.064	0.064	0.487	0.079	0.090	do not pool
Sample 9	0.500	0.355	0.404	0.113	0.123	0.385	0.062	0.068	pool
Sample 10	0.500	0.528	0.544	0.067	0.069	0.530	0.061	0.061	pool
Sample 11	0.500	0.588	0.532	0.043	0.071	0.607	0.048	0.051	pool
Sample 12	0.500	0.570	0.555	0.061	0.063	0.513	0.067	0.088	do not pool
Sample 13	0.500	0.569	0.525	0.058	0.073	0.516	0.080	0.096	do not pool
Sample 14	0.500	0.491	0.464	0.041	0.049	0.501	0.045	0.046	pool
Sample 15	0.500	0.488	0.487	0.090	0.090	0.537	0.098	0.109	do not pool
Sample 16	0.500	0.467	0.455	0.058	0.059	0.480	0.045	0.047	pool
Sample 17	0.500	0.432	0.432	0.048	0.048	0.404	0.048	0.056	do not pool
Sample 18	0.500	0.519	0.506	0.040	0.042	0.516	0.061	0.061	do not pool
Sample 19	0.500	0.493	0.469	0.098	0.101	0.482	0.042	0.043	pool
Sample 20	0.500	0.588	0.554	0.054	0.063	0.620	0.056	0.064	do not pool

Note.—In this table H means headcount, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively. Table 2 column “Survey Map (method 1)” shows how the MSE’s in this table are calculated.

Table 6: Results of the Survey Map Method: Rural Shabwah

	True Value	Survey Value	Non-Pooled			Pooled			Survey Map
	H_c	H_s	\hat{H}_s^{np}	$\sqrt{\text{var}(\hat{H}_s^{np})}$	$\sqrt{MSE_s^{np}}$	\hat{H}_s^p	$\sqrt{\text{var}(\hat{H}_s^p)}$	$\sqrt{MSE_s^p}$	
Sample 1	0.527	0.592	0.526	0.060	0.090	0.542	0.056	0.076	pool
Sample 2	0.527	0.528	0.593	0.052	0.083	0.525	0.059	0.059	pool
Sample 3	0.527	0.606	0.537	0.049	0.084	0.601	0.045	0.045	pool
Sample 4	0.527	0.453	0.445	0.068	0.068	0.443	0.059	0.059	pool
Sample 5	0.527	0.551	0.553	0.047	0.047	0.568	0.052	0.054	do not pool
Sample 6	0.527	0.492	0.492	0.050	0.050	0.492	0.049	0.049	pool
Sample 7	0.527	0.449	0.430	0.046	0.050	0.439	0.057	0.058	do not pool
Sample 8	0.527	0.366	0.315	0.061	0.079	0.356	0.062	0.063	pool
Sample 9	0.527	0.462	0.484	0.055	0.059	0.510	0.051	0.070	do not pool
Sample 10	0.527	0.459	0.385	0.065	0.099	0.403	0.045	0.072	pool
Sample 11	0.527	0.681	0.652	0.046	0.055	0.622	0.053	0.079	do not pool
Sample 12	0.527	0.463	0.403	0.051	0.079	0.421	0.065	0.077	pool
Sample 13	0.527	0.564	0.538	0.055	0.061	0.451	0.056	0.126	do not pool
Sample 14	0.527	0.577	0.478	0.049	0.110	0.488	0.039	0.097	pool
Sample 15	0.527	0.512	0.574	0.048	0.078	0.493	0.066	0.069	pool
Sample 16	0.527	0.514	0.561	0.036	0.059	0.587	0.042	0.085	do not pool
Sample 17	0.527	0.463	0.453	0.054	0.055	0.509	0.048	0.066	do not pool
Sample 18	0.527	0.607	0.601	0.044	0.045	0.589	0.052	0.056	do not pool
Sample 19	0.527	0.574	0.571	0.058	0.058	0.594	0.044	0.049	pool
Sample 20	0.527	0.640	0.630	0.069	0.069	0.607	0.044	0.055	pool

Note.—In this table H means headcount, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively. Table 2 column “Survey Map (method 1)” shows how the MSE’s in this table are calculated.

Table 7: Results of the compare MSE Method: Rural Abyan

	True Value	Survey Value	Non-Pooled			Pooled			compare MSE
	H_c	H_s	\hat{H}_c^{np}	$\sqrt{\text{var}(\hat{H}_c^{np})}$	$\sqrt{MSE_c^{np}}$	\hat{H}_c^p	$\sqrt{\text{var}(\hat{H}_c^p)}$	$\sqrt{MSE_c^p}$	
Sample 1	0.500	0.522	0.532	0.015	0.018	0.508	0.032	0.035	do not pool
Sample 2	0.500	0.531	0.508	0.015	0.028	0.492	0.021	0.044	do not pool
Sample 3	0.500	0.472	0.492	0.027	0.033	0.500	0.014	0.031	pool
Sample 4	0.500	0.534	0.519	0.022	0.027	0.474	0.051	0.079	do not pool
Sample 5	0.500	0.481	0.468	0.029	0.032	0.539	0.023	0.062	do not pool
Sample 6	0.500	0.446	0.434	0.033	0.035	0.427	0.011	0.021	pool
Sample 7	0.500	0.524	0.522	0.033	0.033	0.557	0.053	0.063	do not pool
Sample 8	0.500	0.444	0.407	0.050	0.063	0.445	0.061	0.061	pool
Sample 9	0.500	0.355	0.419	0.093	0.113	0.393	0.046	0.060	pool
Sample 10	0.500	0.528	0.571	0.038	0.057	0.510	0.032	0.036	pool
Sample 11	0.500	0.588	0.519	0.029	0.075	0.575	0.026	0.029	pool
Sample 12	0.500	0.570	0.526	0.040	0.059	0.449	0.055	0.133	do not pool
Sample 13	0.500	0.569	0.516	0.030	0.061	0.546	0.065	0.069	do not pool
Sample 14	0.500	0.491	0.468	0.022	0.031	0.490	0.011	0.011	pool
Sample 15	0.500	0.488	0.497	0.071	0.071	0.539	0.070	0.086	do not pool
Sample 16	0.500	0.467	0.439	0.040	0.049	0.488	0.020	0.029	pool
Sample 17	0.500	0.432	0.494	0.031	0.070	0.457	0.021	0.032	pool
Sample 18	0.500	0.519	0.494	0.015	0.029	0.476	0.034	0.055	do not pool
Sample 19	0.500	0.493	0.494	0.079	0.079	0.508	0.023	0.028	pool
Sample 20	0.500	0.588	0.593	0.028	0.028	0.608	0.027	0.033	do not pool

Note.—In this table H means headcount, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively. Table 2 column “compare MSE (method 3)” shows how the MSE’s in this table are calculated.

Table 8: Results of the compare MSE Method: Rural Shabwah

	True Value	Survey Value	Non-Pooled			Pooled			compare MSE
	H_c	H_s	\hat{H}_c^{np}	$\sqrt{\text{var}(\hat{H}_c^{np})}$	$\sqrt{MSE_c^{np}}$	\hat{H}_c^p	$\sqrt{\text{var}(\hat{H}_c^p)}$	$\sqrt{MSE_c^p}$	
Sample 1	0.527	0.592	0.550	0.041	0.059	0.549	0.031	0.053	pool
Sample 2	0.527	0.528	0.576	0.032	0.058	0.534	0.019	0.020	pool
Sample 3	0.527	0.606	0.607	0.021	0.021	0.578	0.016	0.032	do not pool
Sample 4	0.527	0.453	0.438	0.051	0.053	0.466	0.032	0.035	pool
Sample 5	0.527	0.551	0.597	0.021	0.050	0.544	0.018	0.020	pool
Sample 6	0.527	0.492	0.497	0.027	0.027	0.490	0.010	0.010	pool
Sample 7	0.527	0.449	0.440	0.022	0.024	0.491	0.027	0.049	do not pool
Sample 8	0.527	0.366	0.371	0.051	0.051	0.426	0.065	0.089	do not pool
Sample 9	0.527	0.462	0.511	0.035	0.060	0.512	0.028	0.057	pool
Sample 10	0.527	0.459	0.390	0.053	0.087	0.408	0.028	0.059	pool
Sample 11	0.527	0.681	0.652	0.023	0.037	0.593	0.027	0.093	do not pool
Sample 12	0.527	0.463	0.473	0.019	0.021	0.436	0.051	0.057	do not pool
Sample 13	0.527	0.564	0.553	0.034	0.036	0.444	0.037	0.125	do not pool
Sample 14	0.527	0.577	0.523	0.016	0.057	0.507	0.011	0.071	do not pool
Sample 15	0.527	0.512	0.532	0.013	0.024	0.491	0.040	0.045	do not pool
Sample 16	0.527	0.514	0.524	0.010	0.014	0.525	0.018	0.021	do not pool
Sample 17	0.527	0.463	0.471	0.028	0.029	0.459	0.019	0.019	pool
Sample 18	0.527	0.607	0.570	0.024	0.045	0.552	0.024	0.061	do not pool
Sample 19	0.527	0.574	0.537	0.036	0.051	0.599	0.018	0.031	pool
Sample 20	0.527	0.640	0.600	0.048	0.063	0.611	0.020	0.035	pool

Note.—In this table H means headcount, var means variance, MSE means mean square error, \hat{x} means the estimate of x , p and np in the superscript means pooled and non-pooled respectively and c and s in the subscript means census and survey respectively. Table 2 column “compare MSE (method 3)” shows how the MSE’s in this table are calculated.

Table 9: Comparison of the Methods: Rural Abyan

	True Value H_c	Survey Value H_s	Minimize MSE	Survey Map Method	Compare Value Method	compare MSE Method
1 (Sample 19)	0.500	0.493	pool	pool	pool	pool
2 (Sample 14)	0.500	0.491	pool	pool	pool	pool
3 (Sample 15)	0.500	0.488	do not pool	do not pool	do not pool	do not pool
4 (Sample 5)	0.500	0.481	do not pool	do not pool	do not pool	do not pool
5 (Sample 18)	0.500	0.519	do not pool	do not pool	<i>pool</i>	do not pool
6 (Sample 1)	0.500	0.522	pool	<i>do not pool</i>	pool	<i>do not pool</i>
7 (Sample 7)	0.500	0.524	do not pool	do not pool	do not pool	do not pool
8 (Sample 10)	0.500	0.528	pool	pool	<i>do not pool</i>	pool
9 (Sample 3)	0.500	0.472	pool	<i>do not pool</i>	pool	pool
10 (Sample 2)	0.500	0.531	do not pool	do not pool	<i>pool</i>	do not pool
Successful rate				80%	70%	90%
11 (Sample 16)	0.500	0.467	pool	pool	do not pool	pool
12 (Sample 4)	0.500	0.534	do not pool	do not pool	do not pool	do not pool
13 (Sample 6)	0.500	0.446	do not pool	<i>pool</i>	<i>pool</i>	<i>pool</i>
14 (Sample 8)	0.500	0.444	pool	<i>do not pool</i>	<i>do not pool</i>	pool
15 (Sample 17)	0.500	0.432	do not pool	do not pool	do not pool	<i>pool</i>
16 (Sample 13)	0.500	0.569	do not pool	do not pool	<i>pool</i>	do not pool
17 (Sample 12)	0.500	0.570	do not pool	do not pool	do not pool	do not pool
18 (Sample 11)	0.500	0.588	do not pool	<i>pool</i>	do not pool	<i>pool</i>
19 (Sample 20)	0.500	0.588	do not pool	do not pool	<i>pool</i>	do not pool
20 (Sample 9)	0.500	0.355	pool	pool	pool	pool
Successful rate				75%	65%	80%

Table 10: Comparison of the Methods: Rural Shabwah

	True Value H_c	Survey Value H_s	Minimize MSE	Survey Map Method	Compare Value Method	compare MSE Method
1 (Sample 2)	0.527	0.528	pool	pool	<i>do not pool</i>	pool
2 (Sample 16)	0.527	0.514	do not pool	do not pool	<i>pool</i>	do not pool
3 (Sample 15)	0.527	0.512	do not pool	<i>pool</i>	do not pool	do not pool
4 (Sample 5)	0.527	0.551	pool	<i>do not pool</i>	<i>do not pool</i>	pool
5 (Sample 6)	0.527	0.492	pool	pool	pool	pool
6 (Sample 13)	0.527	0.564	do not pool	do not pool	do not pool	do not pool
7 (Sample 19)	0.527	0.574	do not pool	<i>pool</i>	do not pool	<i>pool</i>
8 (Sample 14)	0.527	0.577	do not pool	<i>pool</i>	<i>pool</i>	do not pool
9 (Sample 17)	0.527	0.463	do not pool	do not pool	<i>pool</i>	<i>pool</i>
10 (Sample 12)	0.527	0.463	do not pool	<i>pool</i>	do not pool	do not pool
Successful rate				50%	50%	80%
11 (Sample 9)	0.527	0.462	pool	<i>do not pool</i>	pool	pool
12 (Sample 1)	0.527	0.592	pool	pool	pool	pool
13 (Sample 10)	0.527	0.459	pool	pool	pool	pool
14 (Sample 4)	0.527	0.453	pool	pool	pool	pool
15 (Sample 7)	0.527	0.449	pool	<i>do not pool</i>	<i>do not pool</i>	<i>do not pool</i>
16 (Sample 3)	0.527	0.606	pool	pool	pool	<i>do not pool</i>
17 (Sample 18)	0.527	0.607	pool	<i>do not pool</i>	pool	<i>do not pool</i>
18 (Sample 20)	0.527	0.640	pool	pool	pool	pool
19 (Sample 11)	0.527	0.681	pool	<i>do not pool</i>	<i>do not pool</i>	<i>do not pool</i>
20 (Sample 8)	0.527	0.366	pool	pool	<i>do not pool</i>	<i>do not pool</i>
Successful rate				55%	60%	65%

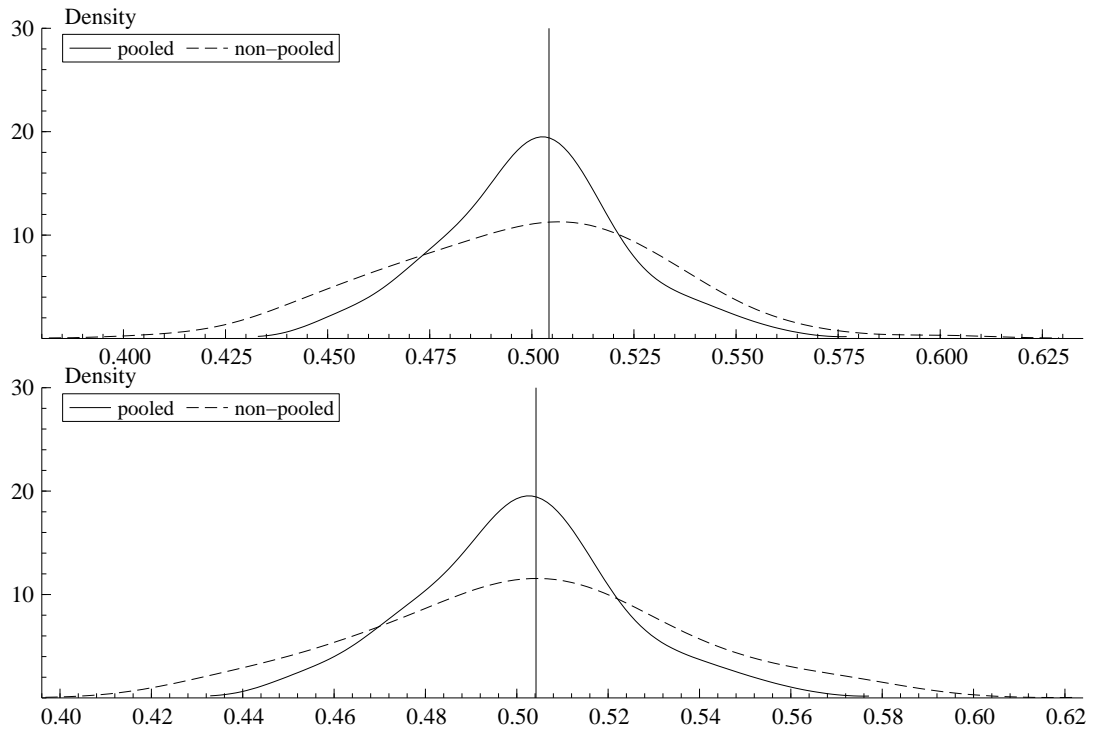


Figure 1: All the parameters are the same in the two strata, $E(x_1) = E(x_2) = 9.1$, $\text{Var}(x_1) = \text{Var}(x_2) = 0.22$, $\beta_1 = \beta_2 = 1.24$, $\sigma_1 = \sigma_2 = 0.5$, poverty line= 80000, $N_1 = N_2 = 100$ and $N_{c1} = N_{c2} = 10000$

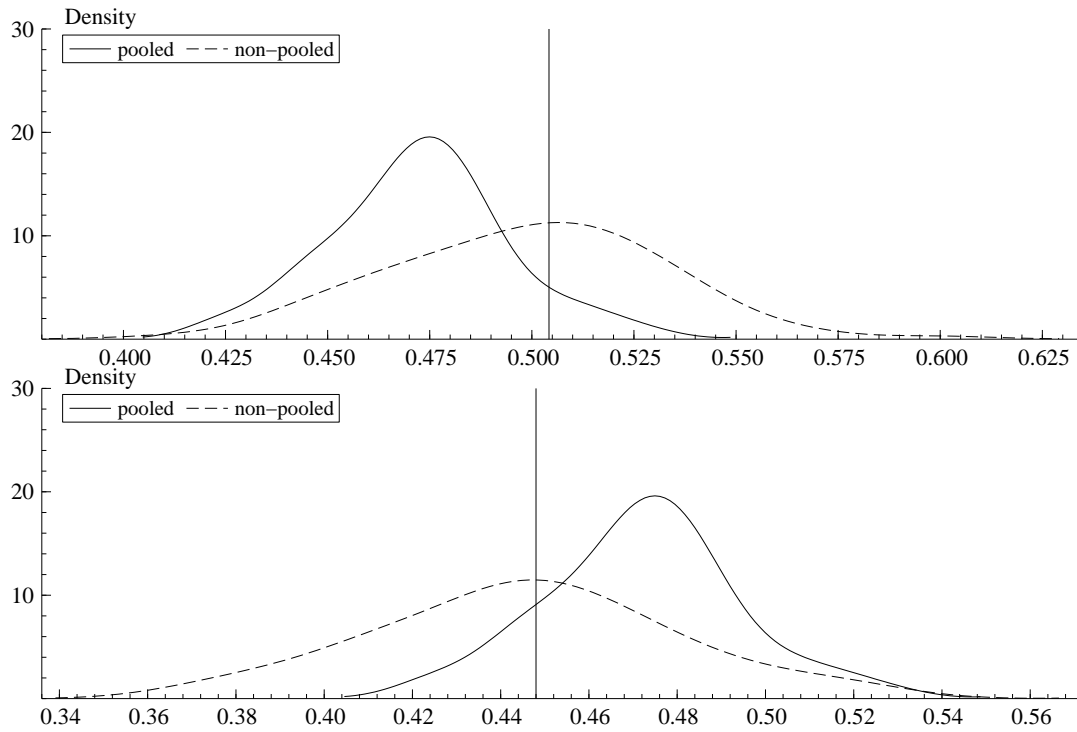


Figure 2: $E(x_1) = E(x_2) = 9.1$, $\text{Var}(x_1) = \text{Var}(x_2) = 0.22$, $\beta_1 = 1.24$, $\beta_2 = 1.249$, $\sigma_1 = \sigma_2 = 0.5$, poverty line = 80000, $N_1 = N_2 = 100$ and $N_{c1} = N_{c2} = 10000$

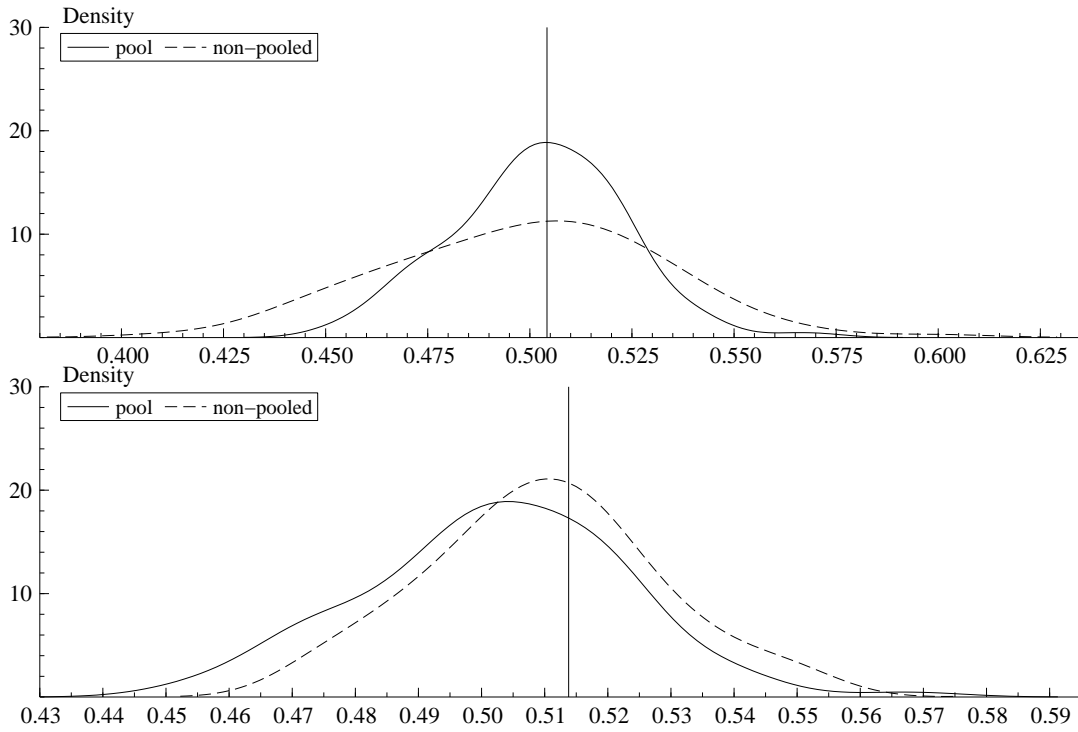


Figure 3: $E(x_1) = E(x_2) = 9.1$, $\text{Var}(x_1) = \text{Var}(x_2) = 0.22$, $\beta_1 = \beta_2 = 1.24$, $\sigma_1 = 0.5$, $\sigma_2 = 0.15$, poverty line = 80000, $N_1 = N_2 = 100$ and $N_{c1} = N_{c2} = 10000$

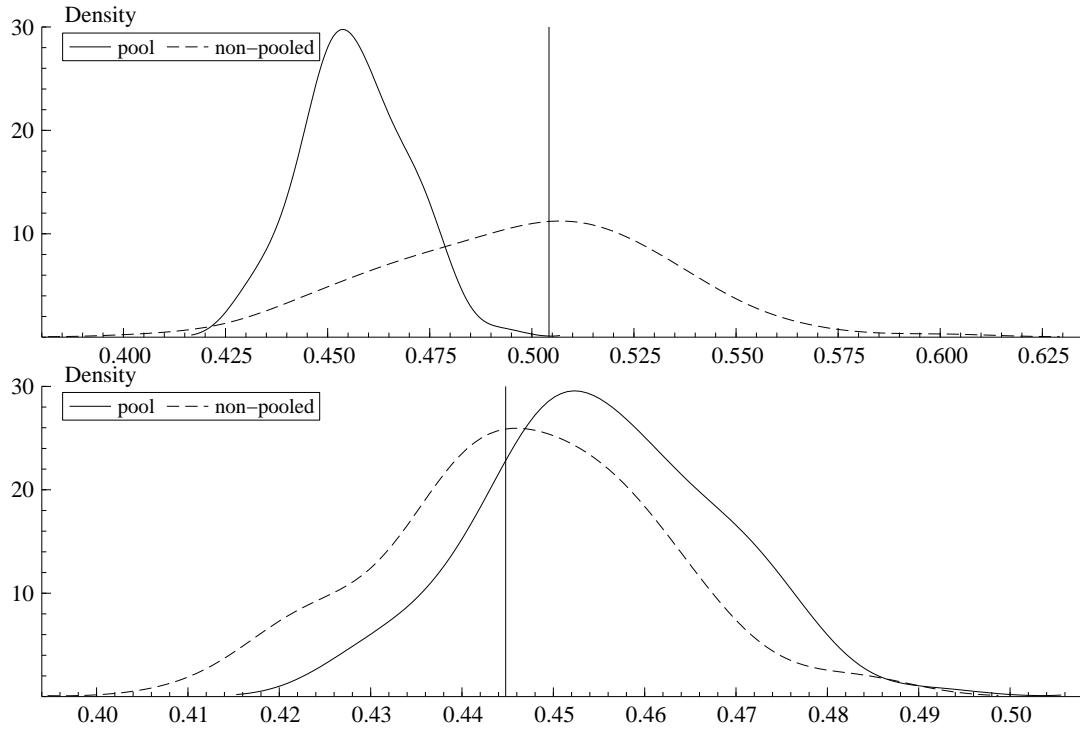


Figure 4: $E(x_1) = E(x_2) = 9.1$, $\text{Var}(x_1) = \text{Var}(x_2) = 0.22$, $\beta_1 = 1.24$, $\beta_2 = 1.249$, $\sigma_1 = \sigma_2 = 0.5$, poverty line = 80000, $N_1 = 100$, $N_2 = 500$, $N_{c1} = 10000$ and $N_{c2} = 50000$